Let $\Omega \subset \mathbb{R}^2$ be a domain with a hole $\omega$. In the domain $A = \Omega \setminus \omega$ consider a class $J$ of complex valued maps having degrees 1 and 1 on $\partial \Omega, \partial \omega$ respectively.

In a joint work with P. Mironescu we show that if $\text{cap}(A) \geq \pi$ (subcritical domain), minimizers of the Ginzburg-Landau energy

$$E_\kappa(u) = \frac{1}{2} \int_\Omega \left( |\nabla u|^2 + \frac{1}{2\kappa^2}(|u|^2 - 1)^2 \right) dx$$

exist for each $\kappa$. They are vortexless and converge in $H^1(A)$ to a minimizing $S^1$-valued harmonic map as $\kappa \to 0$. When $\text{cap}(A) < \pi$ (supercritical domain), for small $\kappa$, we prove that the minimizing sequences/minimizers develop exactly two vortices—a vortex of degree 1 near $\partial \Omega$ and a vortex of degree $-1$ near $\partial \omega$. It was conjectured that the global minimizers do not exist for small $\kappa$.

In a subsequent work with D. Golovaty and V. Rybalko this conjecture was proved. It was shown that, when $\text{cap}(A) < \pi$, there exists a finite threshold value $\kappa_1$ of the Ginzburg-Landau parameter such that the minimum of $E_\kappa$ is not attained in $J$ when $\kappa > \kappa_1$, while it is attained when $\kappa < \kappa_1$. No standard elliptic estimates worked here and our proof is based on an introduction of an auxiliary linear problem which allows for a sufficiently tight explicit energy estimate.