TEMPLE UNIVERSITY

Department of Mathematics

Analysis Seminar

Room 617 Wachman Hall

Monday, November 4, 2019, 2:40 p.m.

Elastic binodal in Calculus of Variations

A blackboard talk at the graduate student level.

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Abstract: The Cauchy-Born principle in physics says that macroscopic affine deformations cause microscopic affine deformations. Mathematically this principle can be formulated in the language of Calculus of Variations: $y(x) = F_0 x$ is the minimizer of the (energy) functional

$$E[y] = \int_\Omega W(\nabla y(x))dx \qquad (\Omega \subset \mathbf{R}^d - \text{a Lipschitz domain})$$

among all Lipschitz functions $y : \Omega \to \mathbf{R}^m$, such that $y(x) = F_0 x$ on $\partial\Omega$. In this form the Cauchy-Born principle can be viewed as a version of Jensen's inequality for convex functions:

$$\frac{1}{|\Omega|}\int_\Omega W(\nabla y(x))dx \geq W(F_0) = W\left(\frac{1}{|\Omega|}\int_\Omega \nabla y(x)dx\right),$$

since $y(x) = F_0 x$ on $\partial\Omega$. When the above inequality holds, we say that $W(F)$ is *quasiconvex* at $F_0 \in \mathbf{R}^{m\times d}$. The boundary of the set of points of quasiconvexity is called the *elastic binodal.* When quasiconvexity fails, the gradients of minimizers of $E[y]$ can become discontinuous or even cease to exist, while minimizing sequences develop fine scale oscillations that people call the *microstructure.*

In this talk I will discuss my joint work with Lev Truskinovsky, aiming to understand when and why such spontaneous discontinuities and microstructures form. This lecture is geared towards graduate students and is meant to be widely accessible.