TEMPLE UNIVERSITY

Department of Mathematics

Analysis Seminar

Zoom meeting

Monday, November 29 2021, 2:30 p.m.

The Cauchy–Szegő projection and its commutator for domains in $\mathbb{C}^n$ with minimal smoothness

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Abstract: Let $D \subset \mathbb{C}^n$ be a bounded, strongly pseudoconvex domain whose boundary bD satisfies the minimal regularity condition of class C^2. A 2017 result of Lanzani & Stein states that the Cauchy–Szegő projection $\mathcal{S}_\omega$ defined with respect to any *Leray Levi-like* measure ω is bounded in $L^p(bD, \omega)$ for any $1 < p < \infty$. (We point out that for this class of domains, induced Lebesgue measure is Leray Levi-like.) Here we show that $\mathcal{S}_\omega$ is in fact bounded in $L^p(bD, \Omega_p)$ for any $1 < p < \infty$ and for any Ω_p in the optimal class of A_p measures, that is $\Omega_p = \psi_p \sigma$ where ψ_p is a Muckenhoupt A_p-weight and σ is induced Lebesgue measure. As an application, we characterize boundedness and compactness in $L^p(bD, \Omega_p)$ for any $1 < p < \infty$ and for any A_p measure Ω_p, of the commutator $[b, \mathcal{S}_\omega]$ for any Leray Levi-like measure ω. We next introduce the notion of holomorphic Hardy spaces for A_p measures, $1 < p < \infty$, and we characterize boundedness and compactness in $L^2(bD, \Omega_2)$ of the commutator $[b, \mathcal{S}_{\Omega_2}]$ of the Cauchy–Szegő projection defined with respect to any A_2 measure Ω_2. Earlier closely related results rely upon an asymptotic expansion, and subsequent pointwise estimates of the Cauchy–Szegő kernel, but these are unavailable in the settings of minimal regularity of bD; at the same time, newer techniques introduced by Lanzani & Stein to deal with the setting of minimal regularity are not applicable to A_p measures that are not Leray Levi-like. It turns out that the method of extrapolation is an appropriate replacement for the missing tools.

This is joint work with Xuan Thinh Duong (Macquarie University), Ji Li (Macquarie University) and Brett Wick (Washington University in St. Louis).