The $L^p$-boundedness of the Riesz transform on graphs and Riemannian manifolds

by Joseph Feneuil
Temple University

Abstract: The Riesz transform $\nabla \Delta^{-1/2}$ on $\mathbb{R}^n$ is bounded on $L^p$ for all $p \in (1, +\infty)$. This well known fact can quickly be proved by using the Fourier transform. Strichartz asked then whether this property is transmitted to Riemannian manifold, more exactly, what are the geometric conditions needed on our manifold to get the boundedness of the Riesz transform.

We shall present (part of) the literature on the topic, including the results of the speaker (together with Li Chen, Thierry Coulhon, and Emmanuel Russ) on fractal-like spaces. We shall also talk about the case of graphs, that can be seen as discrete version of Riemannian manifolds, which will allow us to give concrete examples of application of our work.

If time permits, we will provide equivalent statements for an assumption frequently met when working on graphs (which implies $L^2$-analyticity of the Markov operator). In particular, we will see a way to weaken this assumption to $L^2$-analyticity.