

# TEMPLE UNIVERSITY

Department of Mathematics

## Analysis Seminar

Zoom meeting

Monday, February 14 2022, 2:30 p.m.

*Global regularity and solvability properties for left-invariant operators on compact Lie groups*

by Gabriel Araújo

University of São Paulo-São Carlos

Abstract: Linear PDOs can act on various (generalized) function spaces, provided their coefficients are sufficiently regular: smooth, real-analytic and/or Gevrey spaces, as well as their generalized counterparts, just to name a few. It may then be of interest to establish properties of regularity and solvability (either local or global; several flavors of hypoellipticity; properties of the associated cohomology spaces for systems; and so on) of such PDOs in some of these spaces, sometimes providing radically different answers depending on the space under study.

Some results in the literature, however, establish relationships among such properties for whole classes of operators:

– Greenfield (1972) proved that for operators with constant coefficients on tori global hypoellipticity implies global analytic-hypoellipticity.

– Dealing with differential complexes associated to locally integrable structures, Caetano and Cordaro (2011) proved that if in a given degree the complex is locally solvable in the smooth setup then it is also locally solvable in the Gevrey setup (same degree), while Ragnette (2019), using similar methods, relates these with local solvability in the sense of Gevrey ultradistributions.

– Still dealing with locally integrable structures, Malaspina and Nicola (2014) conjecture another connection between smooth and Gevrey local solvability (a kind of converse to the result of Caetano and Cordaro), which is currently open except for a few cases.

– In a joint work with Cordaro (2019) on analytic structures we connect (in a few particular cases) local solvability in the smooth sense with a property called *semi-local analytic solvability*.

Here we are interested in global properties for systems of left-invariant differential operators on compact Lie groups: regularity properties, properties on the closedness of the range and dimension of cohomology spaces for complexes, when acting on various function spaces. Extending the methods of Greenfield and Wallach (1973) to systems, we obtain abstract characterizations for these properties and use them to derive some generalizations of Greenfield's result, as well as global versions of the result of Caetano and Cordaro for left-invariant involutive structures.