TEMPLE UNIVERSITY

Department of Mathematics

Analysis Seminar

Room 617 Wachman Hall

Monday, February 20th, 2023, 2:30 p.m.

Inverse Iteration for the Monge-Ampère Eigenvalue Problem

by Farhan Abedin

Lafayette College

Abstract: I will present an iterative method for solving the Monge-Ampère eigenvalue problem: given a bounded, convex domain $\Omega \subset \mathbb{R}^n$, find a convex function $u \in C^2(\Omega) \cap C(\overline{\Omega})$ and a positive number λ satisfying

$$\begin{cases} \det D^2 u = \lambda |u|^n & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

By a result of P.-L. Lions, there exists a unique eigenvalue $\lambda = \lambda_{MA}(\emptyset) > 0$ for which this problem has a solution. Furthermore, all eigenfunctions u are positive multiples of each other. In recent work with Jun Kitagawa (Michigan State University), we develop an iterative method which generates a sequence of convex functions $\{u_k\}_{k=0}^{\infty}$ converging to a non-trivial solution of the Monge-Ampère eigenvalue problem. We also show that $\lim_{k\to\infty} R(u_k, \emptyset) = \lambda_{MA}(\emptyset)$, where the Rayleigh quotient $R(v)$ is defined as

$$R(v, \emptyset) := \frac{\int_\Omega |v| \, \det D^2 v}{\int_\Omega |v|^{n+1}}.$$

Our method converges for a large class of initial choices u_0 that can be constructed explicitly, and does not rely on prior knowledge of the eigenvalue $\lambda_{MA}(\Omega)$. I will also discuss other relevant iterative methods in the literature that motivated our work.