Abstract: We discuss a kind of weak-type inequality for the Hardy-Littlewood maximal operator and Calderón-Zygmund singular integral operators that was first studied by Muckenhoupt and Wheeden and later by Sawyer. This formulation treats the weight for the image space as a multiplier, rather than a measure, leading to fundamentally different behavior; in particular, as shown by Muckenhoupt and Wheeden, the class of weights characterizing such inequalities is strictly larger than $A_p$. In this talk, I will discuss quantitative estimates obtained for $A_p$ weights, $p > 1$, that generalize those results obtained by Cruz-Uribe, Isralowitz, Moen, Pott and Rivera-Ríos for $p = 1$, both in the scalar and matrix weighted setting. I will also discuss recent work on the characterization of those weights for which these inequalities hold for the maximal operator.