The Neumann problem for symmetric higher order elliptic differential equations

by Ariel E. Barton
University of Arkansas

Abstract: The second order differential equation $\nabla \cdot A \nabla u = 0$ has been studied extensively. It is well known that, if the coefficients $A$ are real-valued, symmetric, and constant along the vertical coordinate (and merely bounded measurable in the horizontal coordinates), then the Dirichlet problem with boundary data in $L^q$ or $W^{1,p}$, and the Neumann problem with boundary data in $L^p$, are well-posed in the half-space, provided $2 - \varepsilon < q < \infty$ and $1 < p < 2 + \varepsilon$.

It is also known that the Neumann problem for the biharmonic operator $\Delta^2$ in a Lipschitz domain in $\mathbb{R}^d$ is well posed for boundary data in $L^p$, $\max(1, p_d - \varepsilon) < p < 2 + \varepsilon$, where $p_d = \frac{2(d-1)}{d+1}$ depends on the ambient dimension $d$.

In this talk we will discuss recent well posedness results for the Neumann problem, in the half-space, for higher-order equations of the form $\nabla^m \cdot A \nabla^m u = 0$, where the coefficients $A$ are real symmetric (or complex self-adjoint) and vertically constant.