GTA: Philadelphia 2022 Title and Abstract List

Keynote Talks

Counting Pairs of Saddle Connections ∗

Jayadev Athreya, University of Washington

We show that for almost every translation surface the number of pairs of saddle connections with bounded virtual area has asymptotic growth like cR^2 where the constant c depends only on the area and the connected component of the stratum. The proof techniques combine classical results for counting saddle connections with the crucial result that the Siegel-Veech transform is in L^2. In order to capture information about pairs of saddle connections, we consider pairs with bounded virtual area since the set of such pairs can be approximated by a fibered set which is equivariant under geodesic flow. In the case of lattice surfaces, small virtual area is equivalent to counting parallel pairs of saddle connections, which also have a quadratic growth of cR^2 where c depends in this case on the given lattice surface. This is joint work with Samantha Fairchild and Howard Masur.

Enumerative geometry, quantum cohomology, and beyond

Linda Chen, Swarthmore College

Enumerative geometry is the art of counting geometric objects satisfying various conditions. Indeed, when Hilbert listed 23 important unsolved problems for the twentieth century, his Fifteenth Problem was to understand the methods developed by nineteenth century algebraic geometers. I will describe some underlying structures that encode the numbers. For example, a breakthrough in the 1990's inspired by physics gave surprising and beautiful answers to classical enumerative geometry problems such as the enumeration of degree d rational plane curves passing through $3d - 1$ general points. I will also discuss some recent developments, including applications as well as work on equivariant and K-theoretic extensions of these structures.

Floer theoretic invariants of three-manifolds and their localization

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Francesco Lin, Columbia University

The localization theorem in algebraic topology is a powerful tool which provides information about the equivariant cohomology of a space in terms of the fixed point set of the action. It turns out that this sort of techniques can also be applied to obtain information about Floer homologies in low-dimensions (which are topological invariants generally very challenging to compute) leading to interesting results in topology and geometry. I will discuss background concepts, motivating questions, and applications; I will conclude by describing a concrete computation (joint work with M. Miller Eismeier) of the localization of some of these Floer homologies in terms of the classical intersection ring of the manifold.

Isolated points on curves

Bianca Viray, University of Washington

Let C be an algebraic curve over $\mathbb{Q}$, i.e., a 1-dimensional complex manifold defined by polynomial equations with rational coefficients. A celebrated result of Faltings implies that all algebraic points on C come in families of bounded degree, with finitely many exceptions. These exceptions are known as isolated points. We explore how these isolated points behave in families of curves and deduce consequences for the arithmetic of elliptic curves. This talk is on joint work with A. Bourdon, Ö. Ejder, Y. Liu, and F. Odumodu.

Student Talks

Talks marked with a dagger (†) indicate expository talks. Talks marked with a star ∗ indicate virtual talks.

Admissibility of Groups: An Application of Field Patching (†)

Yael Davidov, Rutgers University - New Brunswick

Similarly to the inverse Galois problem, one can ask if a group G is admissible over a given field F. This is answered in the affirmative if there exists a division algebra with F as its center that contains a maximal subfield that is a Galois extension of F, with Galois group G. We will review admissibility results over the rationals that have been proven by Schacher and Sonn. We will also give some idea as to how one might try to construct division algebras that prove the admissibility of a particular group using crossed product algebras. Finally, we will briefly outline how Harbater, Hartmann and Krashen were able to obtain admissibility criteria for groups over a particular class of fields using field patching techniques.

An Alexander method for infinite-type surfaces

Roberta Shapiro, Georgia Tech

The Alexander method is a combinatorial tool used to determine whether two self-homeomorphisms of a surface are isotopic. This statement was formalized in the case of finite-type surfaces by Farb-Margalit, although the main ideas date back 100 years to the work of Dehn. A version of the Alexander method was proven for infinite-type surfaces by Hernández-Morales-Valdez and Hernández-Hidber. We prove the entire statement of the Alexander method, with a special focus on all infinite-type surfaces. In this talk, we will also discuss several applications of the Alexander method, including verifying relations in the mapping class group, showing that the centralizers of certain twist subgroups of the mapping class group are trivial, and providing a simple basis for the topology of the mapping class group.

The Atiyah-Bott Localization Formula in Enumerative Geometry (†) ∗

Dohoon Kim, University of Maryland

In this talk, we first introduce equivariant cohomology, which is a cohomology theory that applies to spaces with a group action. One of the most powerful results in equivariant cohomology is the Atiyah-Bott localization formula: when we have a torus action on a compact, oriented manifold, the localization formula allows us to calculate the integral of an equivariant class as a certain sum over the fixed points. Such equivariant methods are now an important technique in enumerative geometry, and we will provide an example by using the localization formula to solve Steiner's problem, which asks for the number of conics that are tangent to five fixed general conics.

The Bergman Kernel and Biholomorphic Mappings of Pseudoconvex Domains (†)

Achinta Nandi, Oklahoma State University

In this short talk, we shall discuss work of Charles Fefferman on classifying domains on $\mathbb{C}^n$. Using his celebrated work on smooth extension of biholomorphic mappings between strictly pseudoconvex domains with smooth boundary, we shall discuss some results by Chern -Moser to attach explicit geometric invariants on the boundaries of the domains and thus build a powerful tool to classify domains in higher dimensions.

Combinatorics of Exceptional Sequences of Type $\tilde{A}_n$

Raymond Maresca, Brandeis University

It is known that there are infinitely many exceptional collections of representations of Euclidean quivers. In this talk we will study those of Euclidean type $\tilde{A}_n$ and completely describe them in terms of more geometric combinatorial objects known as strand, chord and arc diagrams. We will put every exceptional collection into a so-called family and show that there are only finitely many families. We will also give geometric descriptions of the behavior of these families via Dehn twists on the annulus and lengthening strands.

On Dehn functions of mapping tori of hyperbolic groups $*$

Qianwen Sun, University of Illinois - Chicago

Dehn functions have attracted a lot of attention in recent years. Named after Max Dehn, a Dehn function is an optimal function that bounds the area of a relation in terms of the defining relators in a finitely presented group. It is also closely connected with algorithmic complexity of the word problem, which is the problem of deciding whether a given word equals to 1 in a finitely presented group. There have been a lot of results about Dehn functions of groups. For example, word-hyperbolic groups, automatic groups, finitely generated nilpotent groups, the fundamental group of a closed Riemannian manifold. In 2010, Martin R. Bridson and Daniel Groves proved that the mapping torus of a finitely generated free group has a quadratic isoperimetric inequality. We generalize this result to more general hyperbolic groups which can be represented by an ordered tree of free groups. For a Dehn twist of such a group, we show that the mapping torus has a quadratic Dehn function.

Some Detection Results for Link Floer Homolgy

Fraser Binns, Boston College

Link Floer homology is a powerful vector space valued link invariant due to Ozsváth-Szabó. Given such an invariant it is natural to ask; "For a fixed vector space V, which links have V as their link Floer homology?". I will discuss some strong answers to this question in the form of "detection" results. This talk is based on joint work in progress with Subhankar Dey.

Differential Graded Algebra Structures on Minimal Resolutions of Local Rings (†) $*$

Alexis Hardesty, Texas Tech University

Let $(R, \mathfrak{m}, \Bbbk)$ be a regular local ring and let I a perfect ideal of R of grade 3. In 1978, Buchsbaum and Eisenbud showed that a minimal free resolution $F_\bullet$ of R/I has a differential graded (DG) algebra structure. This DG algebra induces a graded algebra structure on $\mathrm{Tor}_\bullet^R(R/I, \Bbbk) = \mathrm{H}_\bullet(F_\bullet \otimes_R \Bbbk)$. By independent results of Weyman and of Avramov, Kustin, and Miller, this graded algebra structure may be classified into one of five distinct classes. Following this classification, Avramov posed the realizability question, asking which of the five classes can occur under different constraints, such as the type and the minimal number of generators of the ideal I. This talk will be a survey of which classes have been constructively realized as well as experimentally realized and detail the speaker's plans to further answer the realizability question.

Distractions and Generic Initial Ideals *

Anna-Rose Wolff, Purdue University

Given a homogeneous ideal I in a polynomial ring S over a field k we construct, with respect to a given monomial ordering, a monomial ideal of S associated to I denoted by D-gin(I). This ideal is constructed by iteratively computing initial ideals and distractions and is strongly stable in any characteristic. This construction has many properties analogous to the generic initial ideal of I but it has better combinatorial properties than gin(I) when char(k)=p.

From Free by Z to Wn by Z (†)

Mackenzie McPike, Tufts University

There is a matrix condition on ϕ to determine when $F_n \rtimes_\phi \mathbb{Z}$ is hyperbolic. Can we characterize when $W_n \rtimes_\phi \mathbb{Z}$ is hyperbolic? W_n is a RACG that is commensurable to F_{n-1}. In general, can we apply what is known about $F_n \rtimes \mathbb{Z}$ to $W_n \rtimes \mathbb{Z}$?

The Geometry of Sextics and Resolvent Problems

Sidhanth Raman, University of California - Irvine

Here's one of the fundamental problems of mathematics: when nature hands you a single variable polynomial, determine its roots in the simplest manner possible. As it turns out, we're really bad at solving this problem. Simple solutions to algebraic equations are often obstructed due to the underlying geometry of these problems. In a very precise sense, via the language of resolvent degree, we don't even know if sextics are more complicated to solve than quintics. In this talk, we will set the stage to discuss resolvent problems and analytic solutions to polynomials arising from uniformizations, and towards the end touch upon original work that indicates sextics are indeed qualitatively more complex than quintics.

A Hedden-Style Conjecture for String Links

Justin Bryant, Wesleyan University

A conjecture (attributed to Hedden) states that the only endomorphisms of the knot concordance group which are induced by satellite operations are the zero homomorphism, the identity homomorphism, and the involution that takes each class to its inverse. In this talk I will discuss a generalization of this conjecture for maps from the string link concordance group to the knot concordance group which are induced by string link infection. In particular, I will focus on the new homomorphisms that can be defined in this setting and homomorphism obstructions which utilize linking information that was not available in the original conjecture.

Higher-Rank Brill-Noether Theory on $\mathbb{P}^2$

Yeqin Liu and Ben Gould, University of Illinois at Chicago

In this talk we will talk about fundamental aspects of the geometry of Brill-Noether loci in moduli space of stable sheaves on $\mathbb{P}^2$, primarily focusing on (non)emptiness and (irr)reducibility. For the first aspect, ew will give an upper bound and a lower bound on the maximal possible h^0 of stable sheaves in terms of the given Chern character and point out the range where we know sharpness. For the second aspect, we will exhibit a large family of both irreducible and reducible Brill-Noether loci, and we shall see that in some sense "most" Brill-Noether loci are reducible. Moduli spaces of sheaves $M(v)$ on a variety X carry Brill-Noether loci $B^k(v) \subseteq M(v)$, the loci of sheaves E where $h^0(X, E) \geq k$, which are closely related to the geometry of vector bundles on X. We'll develop the foundational properties of Brill-Noether loci on $X = \mathbb{P}^2$. Brill-Noether loci have natural determinantal structure which gives them an expected dimension; when $c_1 = 1$, we'll show that all Brill-Noether loci on $\mathbb{P}^2$ are irreducible and of the expected dimension. When $\mu = c_1/r$ is bigger than $1/2$ and not an integer, and additionally $c_2 \gg 0$, we'll show that the Brill-Noether loci are reducible, with components of both expected and larger-than-expected dimension. This is joint work with Woohyung Lee.

Infinite Staircases in Symplectic Embeddings

Nicki Magill, Cornell University

McDuff and Schlenk determined when there is a symplectic embedding of a four-dimensional ellipsoid into a four-dimensional ball. They found that infinitely many obstructions given by Fibonacci numbers affect these embeddings. This talk will focus on determining whether ellipsoidal symplectic embeddings into other four-dimensional targets are given by infinitely or finitely many obstructions. When considering the target of a blow up of the complex projective plane, we will see the answer is related to the Cantor set. One of the major proof strategies used in this work is almost toric fibrations where we will see how different polygons are used to construct embeddings. Some of this is joint work with Dusa McDuff and Morgan Weiler.

Introduction to Galois Descent (†)

Tamar Blanks, Rutgers University - New Brunswick

Given an algebraic object defined over a field—for example, a quadratic form—which algebraic objects become isomorphic to it when we extend scalars to a larger field? It turns out that this question can be answered in a very general way using a process called Galois descent. In this talk I will outline how Galois descent works and describe some examples and corollaries, including a connection to the Brauer group.

An Invitation To Riordan Group Theory (†) $*$

Shakuan Frankson, Howard University

Pascal's triangle and the binomial theorem are essential concepts that we learn about as mathematicians. What if I told you that we were able to generalize the behavior of binomial coefficients and represent them in a matrix? In Riordan group theory, Pascal's triangle is denoted by P=$\frac{1}{1-z}, \frac{z}{1-z}$. What does this mean? The Riordan group is a group of infinite lower-triangular matrices that are defined by two generating functions, g and f. The k^{th} column of each matrix has entries from the generating function gf^k, where f is the multiplier function. Riordan arrays are applicable in many ways, e.g. they can be used to count combinatorial objects and prove combinatorial identities more efficiently. Furthermore, we can look at double Riordan arrays, which have two multiplier functions, f_1 and f_2, that are multiplied alternately to the columns that follow the initial ""g"" column. An open question that arose was the following: can we find an isomorphism between the Riordan group and a subgroup of the double Riordan group? Can a similar mapping be found to the k-Riordan group? Such questions related to the algebraic structure of the group are yet to be answered. This talk is intended to provide an introduction to Riordan group theory and draw possible connections to other areas of mathematics.

The Jones polynomial of a fibered positive link, and the search for knots with minimal almost-alternating diagrams

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Lizzie Buchanan, Dartmouth College

We set out to produce an infinite family of knots that have a minimal (with respect to crossing number) almost-alternating diagram. While working on this problem, we found a new upper bound on the maximum degree of the Jones polynomial of a fibered positive link. In particular, the maximum degree of the Jones polynomial of a fibered positive knot is at most four times the minimum degree. With this result, we complete the classification of all knots of crossing number less than or equal to 12 as positive or not positive.

Knot Floer homology and train track maps

Bareden Reinoso, Boston College

Knot Floer homology is a powerful algebraic invariant of knots, which comes in the form of a bi-graded vector space. When K is a fibered knot, the graded rank of its knot Floer homology is intimately related to the dynamics of a certain surface map associated to the fibration of its complement. Inspired by this idea, in joint work with Ethan Farber and Luya Wang, we use tools from surface dynamics to study the knot Floer homology of fibered knots. Our methods culminate in a proof that the torus knot $T(2,5)$, AKA the cinquefoil, is the only genus-two L-space knot in S^3, and further work in progress reveals analogous results for other knots, in S^3 and beyond. In this talk, I will highlight the crucial role played by train track maps within this circle of ideas, and discuss how knot theorists, Floer homologists, and dynamicists alike can incorporate train track maps into their daily toolkit.

Kudla-Rapoport conjecture at ramified primes $*$

Qiao He, University of Wisconsin Madison

I will talk about a Kudla-Rapoport type formula for the Kramer model of unitary Rapport-Zink space at a ramified prime of arbitrary dimension. The formula is a precise identity between intersection number of special cycles and derivative of local density polynomials, which is a local analogue of the arithmetic Siegel-Weil formula. In the talk, I will highlight the special feature of this formula and sketch a surprisingly simple proof. This is a joint work with Chao Li, Yousheng Shi and Tonghai Yang.

Large Scale Homology

Zihao Liu, Brandeis University

Generally, large scale geometry considers properties of metric spaces that are visible to an observer at a vantage point preceding to infinity. Specifically, with large scale structures, all bounded metric spaces are equivalent to a point, and thus, the focus will be on unbounded spaces, such as the Cayley graph of a finitely generated infinite group. In this talk, I will introduce my recent work about the large scale homology theory by using the affine n-simplex to characterize topology of the boundary (points at infinity) of an unbounded metric space.

Local-global principles for quadratic forms over function fields $*$

Connor Cassady, University of Pennsylvania

The Hasse-Minkowski theorem states that a quadratic form over a global field is isotropic if it is isotropic over all completions of the field. This is one of the first "local-global principles" for quadratic forms and implies that two quadratic forms over a global field are isometric if they are isometric over each completion. Over more general fields, these local-global principles can be phrased in terms of discrete valuations on the field. In this talk, we explore the local-global principles for isotropy and isometry over function fields with respect to various sets of discrete valuations. We will see that over rational function fields, it is "easy" to satisfy the local-global principle for isometry, but hard to satisfy the local-global principle for isotropy. More generally, over finitely generated field extensions of transcendence degree r over an algebraically closed field, we use the 2^r-dimensional counterexample to the local-global principle for isotropy of Auel and Suresh to show that there exist counterexamples of dimension $< 2^r$ with respect to sets of discrete valuations arising naturally from geometric considerations.

Metric Spaces of Arbitrary Finitely-Generated Scaling Groups

Daniel Levitin, University of Wisconsin - Madison

A quasi-isometry between uniformly discrete spaces metric spaces of bounded geometry is *scaling* if it is k-to-1 on finite subsets up to an error term that takes into account the geometry of the set. The collection of k for which scaling self-maps exist is a multiplicative group by composing maps. Scaling maps and the scaling group have been used to prove a variety of quasi-isometric rigidity theorems for groups and spaces. In this talk, I will trace the development of this theory from Whyte's early results on BiLipschitz maps of non-amenable spaces to Genevois and Tessera's rigidity theorem on certain wreath products. I will then sketch my construction of a space with any finitely-generated scaling group.

Milnor's invariants for knots in spherical 3-manifolds

Ryan Stees, Indiana University - Bloomington

In his 1957 paper, John Milnor introduced a collection of invariants for links in the 3-sphere detecting higher-order linking phenomena by studying quotients of the link group by its lower central series. These invariants were later shown to be link concordance invariants, and have since inspired decades of consequential research. In particular, several efforts have been made to extend these invariants to knots in other 3-manifolds. We will describe new constructions of such invariants for knots in spherical 3-manifolds, give some concrete examples, and relate our work to previous efforts. Time permitting, we will fit our work into the larger context of Poincaré embedding type, a universal homotopy invariant of knot concordance.

On a family of pseudo-Anosov braids, and an invitation to train tracks

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Ethan Farber, Boston College

We apply the theory of train tracks to introduce and study a family of positive pseudo-Anosov braids. The train tracks we construct are essentially intervals, allowing us to investigate our braids using classical techniques from 1-dimensional dynamics. Remarkably, 1D invariants recover geometric and dynamical information about these 2D braids. In particular, we find that the braids are parameterized by their fractional Dehn twist coefficient (FDTC), a geometric quantity measuring the rotation at infinity. Moreover, the FDTC structures this family of braids into a dynamical ancestry, wherein new braids are generated from the dynamics of older ones. We hope that this talk serves as an advertisement for train tracks, which sit at the intersection of dynamics, geometry, and algebra.

On Tor-rigidity and the reflexivity of tensor products *

Uyen (Enni) Le, West Virginia University

In 1994, Huneke and Wiegand introduced the Depth Formula, but in 2007, they announced in an erratum that one of the conclusions in the depth formula theorem is flawed due to an incorrect convention for the depth of the zero module. Not until 2019 was the claim proven to be false by Celikbas and Takahashi. In the same year, Celikbas, Matsui, and Sadeghi showed that Tor-rigidity was the extra condition that needed for the depth formula to hold true. In this talk, we determine new conditions under which the depth formula theorem holds true, i.e., if the tensor products of finitely generated modules over hypersurface rings is reflexive, then both of their factors are reflexive. Finally, we will discuss some examples to show why these results are new and necessary.

Pillowcase Homology and Singular Instanton Homology

Kai Smith, Indiana University - Bloomington

Pillowcase homology is a proposed knot homology theory by Hedden, Herald, and Kirk constructed by decomposing a knot into two tangles and counting the intersections of their traceless SU(2) character varieties inside a space called the pillowcase. The motivation behind pillowcase homology is its conjectured relationship with Kronheimer and Mrowka's singular instanton homology. In this talk I will give an overview of the two homology theories and I will present new examples which show the limits of their relationship.

Seiberg-Witten Floer K-Theory and Cyclic Group Actions on Spin 4-Manifolds with Boundary

Ian Montague, Brandeis University

I will outline the construction of a metric-independent $\mathrm{Pin}(2)\widetilde{\times}\mathbb{Z}_m$-equivariant Seiberg-Witten Floer spectrum $\mathrm{SWF}(Y)$ associated to a spin rational homology 3-sphere Y equipped with a spin $\mathbb{Z}_m$-action, as well as equivariant analogues of Manolescu's invariant $\kappa(Y)$, defined as the minima of a certain semi-infinite lattice associated to the equivariant K-theory of $\mathrm{SWF}(Y)$. As an application, I will discuss how these invariants provide bounds on the intersection forms of equivariant spin fillings of Y, as well as obstructions for H-sliceness in spin 4-manifolds.

Symplectic Instanton Knot Homology

David White, North Carolina State University

Motivated in part by the Atiyah-Floer conjecture, there have been a number of constructions of Lagrangian Floer homology invariants for 3-manifolds which are defined in terms of symplectic character varieties arising from Heegaard splittings. We develop a relative variant of one of these homologies, due to H. Horton, for knots in 3-manifolds. The foundational machinery is drawn from knot Floer homology (HFK) and from the extensive literature on the symplectic properties of character varieties with holonomy restrictions.

Twisted Alexander Polynomials and a Conjecture by Dunfield-Friedl-Jackson (†)

Michael Marinelli, CUNY - Graduate Center

The twisted Alexander polynomial (TAP) was discovered in 1991, yet in contrast to the classic Alexander polynomial very little is known about it. A conjecture by Dunfield-Friedl-Jackson in 2012 states that for hyperbolic knots, the TAP related to the holonomy representation is monic exactly when the knot is fibered. In this talk, we will define the twisted Alexander polynomial, formulate Dunfield-Friedl-Jackson's conjecture, and explore a few cases where the conjecture has been proven.

Twisted Trisection

Terrin Warren, University of Georgia

Given two trisected 4-manifolds X and X', we can consider the natural trisection on the connected sum $X \# X'$. We can show that the Dehn twist along the separating 3-sphere is a self-diffeomorphism of the trisection which results in an isotopic trisection. Moreover, the restriction of this twist onto the trisection surface can be seen by a Dehn twist along the separating circle.

Understanding Weight Filtrations via Derived Motivic Measures

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Anubhav Nanavaty, University of California - Irvine

Motivic measures appear in many contexts in algebraic geometry and topology, such as the compactly supported Euler characteristic of complex varieties, or the point counting measure of Fp varieties. After the work of Zakharevich, who constructed a K theory spectrum of the category of varieties, work as been done to lift these motivic measures to maps of K theory spectra (i.e. making them "derived" measures). This talk will outline results in this direction and discuss open questions. In doing so, we will give an outline of some methods for understanding the topology of singular algebraic varieties, such as cohomological descent techniques used in the construction of the Gillet-Soulé weight filtration, and show that these methods allow us to lift measures to K theory spectra, showcasing the conceptual strength of these approaches.
