GSCAGT 2021 Title and Abstract List

Talks marked with a dagger (†) indicate expository talks

3D Mirror Symmetry and the Combinatorics of Cherkis Bow Varieties

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Yiyan Shou, University of North Carolina, Chapel Hill

It is believed that the phenomenon of 3d (N=4) mirror symmetry manifests itself in certain relationships between torus equivariant elliptic characteristic classes of mirror dual pairs of holomorphic symplectic varieties. Thus far, this notion of 3d mirror symmetry for characteristic classes has been proven in some special cases (e.g. hypertoric varieties and cotangent bundles of Grassmannians and full flag varieties) using complicated ad hoc methods, but no general theory is known. We believe that Cherkis bow varieties (which include Nakajima quiver varieties) are the most natural setting for the study of 3d mirror symmetry and that in this setting, the desired relationships among characteristic classes can be obtained through natural combinatorial arguments. This talk aims to give an overview of 3d mirror symmetry for characteristic classes as well as the combinatorial framework for the study of Cherkis bow varieties developed in a recent work with Richard Rimanyi.

Big Pure Mapping Class Groups are Never Perfect

George Domat, University of Utah

We prove that pure mapping class groups of infinite type surfaces are never perfect, i.e., they have non-trivial abelianization. This is in contrast to the result of Powell that pure mapping class groups of finite type surfaces are perfect if the surfaces have genus at least 3. In fact, we show that the abelianization of a particular closed subgroup always contains uncountable direct sums of rationals. To accomplish this we make use of the projection complex machinery of Bestvina, Bromberg, and Fujiwara in order to build quasimorphisms that "see" this non-perfectness.

Botany Questions in Knot Floer Homology

Fraser Binns, Boston College

Knot Floer homology is an invariant of links taking value in the category of vector spaces. Given such an invariant it is natural to ask questions like "can I identify the links with a specified vector space as their knot Floer homology?". This is "the botany question" for knot Floer homology. In this talk I will address various special cases of this problem. This is based on joint work with Subhankar Dey.

The Conley Index for Discrete Dynamical Systems (†)

Isabella Alvarenga, Federal University of ABC

The Conley Index Theory, introduced by on 1978 by Charles Conley, had as a main purpose to generalize the Morse Theory for Dynamical Systems with invariant sets more general then hyperbolic singularities. In this context, the Conley Index give us a homological description of the local dynamic, and thus take into account the topology of the adjacent space. The main goal of this talk is to present the definition of the Conley Index for Discrete Dynamical Systems and its differences when compared to the Conley Index for Continuous Dynamical System. As an example, we present the Conley Index for the discrete dynamical system generated by the tent map.

Constructing Proper Affine Actions

Neza Zager Korenjak, University of Texas at Austin

Using ideas by Danciger-Gúeritaud-Kassel, we construct affine deformations of Fuchsian representations into $SO(2n, 2n-1)$. Through a direct computation of the Margulis invariant for cocycles arising in this way, we identify a family of representations acting properly on $R^{2n,2n-1}$.

Cusps of Hyperbolic 4-Manifolds

Connor Sell, Rice University

The cross-sections of the cusps of a orientable hyperbolic 4-manifold must be flat, compact, orientable 3-manifolds. It is known that all six such 3-manifolds must occur in the cusp of a hyperbolic 4-manifold; it is also known that the 3-torus occurs in every commensurability class of such 4-manifolds. Until recently, it was unknown how frequently most of these cusp types occur, and specifically whether any commensurability classes avoid certain cusp types. In this talk, I will demonstrate that three out of the six are avoided by some classes of arithmetic hyperbolic 4-manifolds.

Dieudonné Theory Via Stacks

Shubhodip Mondal, University of Michigan

In number theory and algebraic geometry, group schemes are objects of fundamental interest with many applications. Certain group schemes can be classified via the classical Dieudonné theory which allows one to study them in a linear algebraic way. In this talk, I will first mention the classical construction of a Dieudonné module. Then I will explain a new topological construction of these Dieudonné modules by using the classifying stack of a group scheme. More precisely, we will see that the second crystalline cohomology of the classifying stack recovers the Dieudonné module. I will discuss similar results in the context of p-divisible groups as well. Lastly, I will discuss analogues of such results in mixed characteristic by using the recently developed prismatic cohomology of Bhatt and Scholze.

Differential Essential Dimension

Man Cheung Tsui, University of Pennsylvania

Using the change of variables $x = y - a/2$, the general quadratic $x^2 + ax + b$ simplifies to the polynomial $y^2 + c$ in one parameter $c = b - a^2/2$. Similarly, a linear change of variables simplifies the general cubic $x^3 + ax^2 + bx + c$ to one of the form $y^3 + dy + d$ in one parameter d. More generally, one would like to simplify a general polynomial using algebraic changes of variables called Tschirnhaus transformations. By introducing the theory of essential dimensions, Joe Buhler and Zinovy Reichstein were able to bound the number of parameters that can be eliminated from a general polynomial using Tschirnhaus transformations. In this talk we show that the number of parameters in a general homogeneous linear differential equation over a field cannot be reduced via gauge transformations over the given field. We do this by introducing an analogue of essential dimension in differential algebra.

Expansion/Contraction Dynamics for Non-Strictly Convex Projective Manifolds

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Theodore Weisman, University of Texas at Austin

Anosov representations are a higher-rank generalization of convex cocompact subgroups of rank-one Lie groups, given in terms of the dynamics of the action of a discrete group on its limit set in an appropriate flag manifold. Work of Danciger, Gueritaud, and Kassel in 2017 established a strong link between Anosov representations and the holonomy representations of compact convex projective manifolds with Gromov-hyperbolic fundamental group. We generalize this relationship to the case where the fundamental group is not necessarily hyperbolic, and show that convex cocompact representations in projective space are characterized by Anosov-like expansion/contraction dynamics on a family of Grassmannians. When the fundamental group is relatively hyperbolic, this has a nice description in terms of an embedding of its Bowditch boundary into a certain quotient of projective space.

Fekete Polynomials, Quadratic Residues, and Arithmetic

Tung Nguyen, University of Chicago/Western University

Fekete polynomials play an important role in the study of special values of L-functions. While their analytic properties are well-recorded in the literature, little is studied about their arithmetic. In this talk, we plan to explain some surprising properties of these polynomials. In particular, we will see that Fekete polynomials contain some rich arithmetic information. This is based on joint work with Jan Minac and Nguyen Duy Tan.

Finite Quotients of Triangle Groups

Frankie Chan, Purdue University

A result from Bridson, Conder, and Reid establishes that two distinct Fuchsian groups are distinguishable by their finite quotients. Starting with a co-compact triangle group, we effectivize upper bounds for the sizes of finding such finite quotients in comparison to other finitely generated Fuchsian groups. We will pay special attention to understanding genus zero Fuchsian groups. This is joint work with Ryan Spitler.

Gerstenhaber Bracket on Hopf Algebra Cohomology

Tekin Karadag, Texas A& M University

We calculate the Gerstenhaber bracket on Hopf algebra and Hochschild cohomologies of the Taft algebra T_p for any integer $p > 2$ which is a nonquasi-triangular Hopf algebra. We show that the bracket is indeed zero on Hopf algebra cohomology of T_p, as in all known quasi-triangular Hopf algebras. This example is the first known bracket computation for a nonquasi-triangular algebra.

Group Trisections and Smoothly Knotted Surfaces

Sarah Blackwell, University of Georgia

A trisection of a 4-manifold induces a Van Kampen cube of fundamental groups coming from the pieces of the trisection, and more surprisingly, vice versa. That is, a Van Kampen cube satisfying a few simple requirements produces a trisection of a 4-manifold. One natural question to ask is whether the same holds for bridge trisections of knotted surfaces in the 4-sphere. In this talk I will introduce this question along with relevant definitions, and briefly describe progress towards a resolution. This is joint work with Rob Kirby, Michael Klug, Vince Longo, and Benjamin Ruppik.

The Images of Polynomials Evaluated over Matrices

Emily Hoopes-Boyd, Kent State University

Let $M_n(F)$ be the ring of $n \times n$ matrices over an infinite field, F. The L'vov-Kaplansky conjecture states that the image of a multilinear polynomial evaluated over $M_n(F)$ is a vector space. This statement is still an open problem, but many partial results have been proven within the last decade. We will consider this problem in a slightly different context; rather than taking the matrix entries to be from an infinite field, we will consider matrices over an algebraically closed skew field, which we will denote by K. We will show that the image of any multilinear polynomial with coefficients from K, evaluated over $M_n(K)$, is $M_n(K)$. We will also prove that any matrix in $M_n(K)$ may be written as the sum of three or fewer elements from the image of any generalized polynomial. In particular, the image of the polynomial $xy - yx$ has some special properties over a variety of matrix rings.

Infinitely Many Knots with Non-Integral Trace

Nicholas Rouse, Rice University

A finite volume hyperbolic 3-manifold can be thought of as the quotient of hyperbolic 3-space by a discrete subgroup of $\mathrm{PSL}(2, \mathbf{C})$. By Mostow-Prasad rigidity the traces of elements of this subgroup are algebraic numbers. These traces are often algebraic integers (for example when the manifold is arithmetic), but there are many examples of subgroups with non-integral trace. We say the manifold has non-integral trace if there is an element of the corresponding group that does. The existence of a non-integral trace has geometric consequences; for example, Bass's theorem implies that a manifold with non-integral trace contains a closed embedded essential surface. I will discuss joint work with Alan Reid constructing infinitely many hyperbolic knot complements in the 3-sphere that have non-integral trace. I will mention some broad details of the proof as well as a few related questions.

Invariance of Knot Lattice Homology

Seppo Niemi-Colvin, Duke University

Links of singularity and generalized algebraic links are ways of constructing three-manifolds and smooth links inside them from algebraic surfaces and curves inside them. Némethi created lattice homology as an invariant for links of normal surface singularities which developed out of computations for Heegaard Floer homology. Later Ozsváth, Stipsicz, and Szabó defined knot lattice homology for generalized algebraic knots, which is known to play a similar role to knot Floer homology and is known to compute knot Floer in some cases. I discuss a proof that knot lattice is an invariant of the smooth knot type, which had been previously suspected but not confirmed.

Kähler Groups and Finitely Generated Groups Acting on Trees

Francisco Nicolás Cardona, Strasbourg University

In this talk I will present some negative results about K¨ahler groups that admit as a normal subgroup a finitely generated torsion-free group acting on a tree. One of the main ingredients of this work is a result of Gromov and Schoen about K¨ahler groups acting on trees. I will briefly recall the construction of the dual tree associated to a simple closed curve in a closed oriented surface of genus $g \geq 2$. This will allows us to study the particular case of K¨ahler groups that admit as a normal subgroup a surface group. More precisely, we will see that if a surface group embeds as a normal subgroup in a K¨ahler group and the conjugation action of the K¨ahler group on the surface group preserves the conjugacy class of a non-trivial element, then the K¨ahler group is virtually given by a direct product, where one factor is a surface group.

Minimal Models of Character Varieties (†)

Yi Wang, University of Pennsylvania

The work of Culler-Shalen utilized the character variety and Thurston's canonical component to detect essential surfaces in hyperbolic 3-manifolds. This work has been extended by several others in discovering the A-polynomial, whose algebraic properties reflect topological properties of detected essential surfaces. When viewed as a scheme, one can approach Thurston's canonical component from an arithmetic point of view by examining the properties of the integral model. In this mostly expository talk I will discuss Culler-Shalen theory, the A-polynomial, minimal models, and the theory of reductions. If there is time I will discuss some speculation and new research regarding these techniques.

Operads, ∞-Categories, and Dendroidal Sets (†)

Elijah Gunther, University of Pennsylvania

Lurie and Joyal developed the use of quasicategories, a type of simplicial set, as a geometric model for ∞-categories. In this model, a 1-simplex represents a morphism and an n-simplex represents the composition of n composable morphisms given by its 1-faces.
Categories can be generalized to operads, which allow for k-ary multimorphisms or operations from k objects to one. As an example, multilinear functions from the product of k vector spaces to a single vector space. Moerdijk and Weiss then introduced dendroidal sets to model ∞-operads. In this talk, I will explain how dendroidal sets give a geometric model of ∞-operads, generalizing both operads and quasicategories. Just as in a category morphisms compose linearly, in an operad they compose in the shape of a tree, with multiple branches coming together into a single trunk. Dendroidal sets are then formed as a union of dendrices which look like trees, each representing a particular sort of composition, analogous to the simplices of a quasicategory.

The Ordered Configuration Space of the Once-Punctured Torus and Secondary Representation Stability

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Nicholas Wawrykow, University of Michigan

Church, Ellenberg, and Farb proved that the homology groups of the ordered configuration space of a connected noncompact orientable manifold stabilize in a representation theoretic sense as the number of points in the configuration grows, with respect to a map that adds each new point “at infinity.” Miller and Wilson showed that there is a secondary representation stability pattern among the unstable homology classes, with respect to adding a pair of orbiting points “near infinity.” In this talk, I will explain how these maps arise, and I will show that the secondary representation stability map is neither free nor eventually zero for the homology groups of the ordered configuration space of the once-punctured torus.

A Polytopal Decomposition of Strata of Translation Surfaces

Bradley Zykoski, University of Michigan

A closed surface can be endowed with a certain locally Euclidean metric structure called a translation surface. Moduli spaces that parametrize such structures are called strata, and there is still much to discover of their global topology. These strata admit a decomposition into finitely many polytopal regions parametrized by certain triangulations of translation surfaces (L^∞ Delaunay triangulations). These regions are adjacent to each other in pathological ways, but it was conjectured by Frankel that these pathologies can be nicely classified. We affirm this conjecture of Frankel, and use the resulting classification to endow strata with an explicit finite cellular structure.

Poncelet Polygons and Some Invariants

Ana Chavez Caliz, The Pennsylvania State University

During his time captive in prison in Saratov (1813-1814), Jean Poncelet developed one of his most important work: ""Traité des propriétés projectives des figures,"" establishing the foundations of projective geometry. One of his most famous results is the Poncelet Porism (which gives rise to a special family of polygons known as Poncelet polygons). What is surprising is that, even though this result has been around for more than 200 years, we still find questions without an answer relating to these families of polygons. During this talk, I will present a few recent results and a sketch of the tools and techniques used to solve these problems. The talk will include a glimpse into some open questions in the field.

Projective Complex Fenchel-Nielsen Coordinates (†)

Rodrigo Dávila Figueroa, Instituto de Matematicas UNAM

The purpose of this work is generalize the Fenchel-Nielsen coordiantes but now for the space of faithful, discrete and purely strong loxodromic representations of surface groups in $SL(3, \mathbb{C})$ and since this space instersects with other represenation spaces ($SL(2, \mathbb{K})$ with $K = \mathbb{R}, \mathbb{C}$, $SL(3, \mathbb{R})$ and $SU(2, 1)$ we want to relate our coordinates with coordinates given in these scenarios.

QI Rigidity of Lattice Products

Josiah Oh, Ohio State University

Schwartz proved quasi-isometric rigidity for non-uniform lattices in rank one Lie groups. Frigerio–Lafont–Sisto later proved QI rigidity for products $\pi_1(M) \times \mathbb{Z}^d$ where M is a complete, non-compact, finite-volume real hyperbolic manifold of dimension at least 3. This talk will cover a recent result on QI rigidity for products $\Lambda \times L$, where Λ is a non-uniform lattice in a rank one Lie group and L is a lattice in a simply connected nilpotent Lie group. Specifically, any finitely generated group quasi-isometric to such a product is, up to some finite noise, an extension of a non-uniform rank one lattice by a nilpotent lattice. Under some extra hypotheses, this extension is nilcentral, a notion we introduce as a generalization of central extensions.

Rank Growth of Elliptic Curves in S_4 Quartic Extensions

Daniel Keliher, Tufts University

Fix an elliptic curve $E : y^2 = x^3 + Ax + B$ and a number field K. If $E(K)$ is the group of K-rational points of E, then the Mordell-Weil Theorem says that $E(K)$ decomposes into a free abelian part, $\mathbb{Z}^{r(E/K)}$ (where $r(E/K) \in \mathbb{N}$ is the rank of E over K) and a finite abelian part. Both components remain objects of active study. In this talk we ask, given an extension of number fields F/K, how do $r(E/K)$ and $r(E/F)$ differ? Restricting ourselves to the case where F is an S_4 quartic extensions of $K = \mathbb{Q}$, we'll show, under mild assumptions, that there are infinitely many S_4 quartic extensions of $\mathbb{Q}$ over which E does not gain rank, i.e. where $r(E/\mathbb{Q}) = r(E/F)$.

Reducedness of Unramified Algebras over Fields

Alapan Mukhopadhyay, University of Michigan

A finite type algebra over a field which is unramified over the field is necessarily reduced. However, there are examples of non-reduced unramified algebras over fields of both characteristic zero (due to Gabber) and positive characteristics. Despite of these examples, we shall show that under suitable graded and local hypothesis, unramified algebras are reduced. The talk is based on a joint work with Karen E. Smith and is available at `https://arxiv.org/abs/2005.05833`.

Rigidity of the Automorphism Group of a Universal Coxeter Group

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Yassine Guerch, Paris-Saclay University

A universal Coxeter group of rank n, denoted by W_n, is the free product of n copies of a cyclic group of order 2. In this talk I will present some rigidity results for the outer automorphism group $Out(W_n)$ of a universal Coxeter group of rank n. Namely, I will sketch a proof that every automorphism of the outer automorphism group of a universal Coxeter group of rank at least 5 is induced by a global conjugation and, more generally, every isomorphism between two finite index subgroups of $Out(W_n)$ with n at least 5 is induced by a global conjugation. These algebraic rigidity results rely on the study of the action of $Out(W_n)$ on the Guirardel-Levitt spine of Outer space of W_n, whose symmetries are exactly described by elements of $Out(W_n)$.

Scaled Homology and Topological Entropy

Zihao Liu, Brandeis University

The topological entropy $h(f)$ of a map $f : M \to M$ is a non-negative real number measuring how much f mixes up the space M. There is an entropy conjecture relating $h(f)$ to $r(f_*)$, the spectral radius of $f_* : H_*(X;\mathbb{R}) \to H_*(X;\mathbb{R})$. The idea is that $h(f)$ should be bounded below by $\log r(f_*)$. One of the known results is that the entropy conjecture holds for the first homology group once M is a compact smooth manifold. My work generalizes the existing results of entropy conjecture, relaxing the restrictions on the compact metric space on which f acts. The entropy conjecture will then hold for the first homology group with lc-homology I defined.

Schur Multipliers and Second Quandle Homology

Dionne Ibarra, George Washington University

We introduce Alexander quandles and show how the second quandle homology of a quasigroup Alexander quandle X is related to Shur multipliers by expressing it in terms of the exterior algebra of X.

Skew Lines on Cubic Threefolds

Yilong Zhang, Ohio State University

Take a pair of skew lines in $\mathbb{P}^3$, one can move one line to intersect the other line at one point. However, if we want the family to be flat, an embedded point should be added at the intersection of the two lines. This leads to the notion of Hilbert scheme as a compactification of the set of all pairs of skew lines. Chen-Coskun-Nollet showed that the Hilbert scheme of skew lines on $\mathbb{P}^3$ is smooth and is isomorphic to the

blowup $Bl_\Delta Sym^2 Gr(2,4)$ of the symmetric product of the Grassmanian along the diagonal, and similarly for projective space of higher dimensions. We generalize this result to smooth cubic hypersurface of $\mathbb{P}^4$. As an application, we characterize the completion of the "root systems" of cubic surfaces as they vary in hyperplane sections of the cubic threefold.

Some Quantum Symmetries of Path Algebras

Amrei Oswald, University of Iowa

Much like group actions formalize the notion of symmetry, Hopf actions of quantum groups formalize the notion of quantum symmetry. We investigate an example of quantum symmetry by studying Hopf actions of $U_q(\mathfrak{b})$ and $U_q(\mathfrak{sl}_2)$ on path algebras. First, we parametrize these actions using linear algebraic data. Then, we attempt to describe the "building blocks" of these actions by viewing them as tensor algebras in the tensor categories $\mathsf{rep}(U_q(\mathfrak{b}))$ and $\mathsf{rep}(U_q(\mathfrak{sl}_2))$. We construct an equivalence between categories of bimodules in $\mathsf{rep}(U_q(\mathfrak{b}))$ (or $\mathsf{rep}(U_q(\mathfrak{sl}_2))$) and a subcategory of certain finite-dimensional representations of associative algebras, explicitly given in terms of quivers with relations.

Structures of Optimal Allowable Sequences and Their Corresponding Configurations Outside General Position

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Rimma Hamalainen, California State University Northridge

In 1982, Peter Ungar proved with the use of allowable sequences that configurations of $2n$ points in a plane that are not all collinear determine at least $2n$ directions. In general position, the configurations that achieve this lower bound consist of regular $2n$-gons and their affine transformations.
Our goal is to understand the structure of realizable allowable sequences that are optimal, i.e. restricted to the total of $2n$ permutations outside of the assumption of general position. Thus we research different configurations that correspond to this lower bound, which are not affine transformations of $2n$-gons. These configurations and their properties are classified based on the numerical partitions of n as well as the structures of their corresponding allowable sequences.
We are particularly interested in configurations built on $\mathbb{Z}^2$ or using the points of the hexagonal lattice. The realizability problem of the optimal allowable sequence is also of great interest of ours and the restrictions imposed on realizability. We tackle this problem from a geometric perspective using tools from combinatorial geometry.

Unknotting 2-Knots with Finger- and Whitney Moves

Benjamin Matthias Ruppik, Max Planck Institute for Mathematics

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This is joint work (arxiv.org/abs/2007.13244) with Jason Joseph, Michael Klug, Hannah Schwartz. Any smoothly knotted 2-sphere in the 4-sphere is regularly homotopic to the unknot. This means that every 2-knot K in S^4 can be obtained by first performing a number of trivial finger moves on the unknot, and then removing the resulting intersection points in pairs via Whitney moves along possibly complicated Whitney discs. We define the Casson-Whitney unknotting number of the 2-knot K as the minimal number of finger moves needed in such a process to arrive at K.

In this talk, I would like to show examples of families of 2-knots (ribbon 2-knots, twist-spun 2-knots) and tell you why they are interesting. Then we can study algebraic lower bounds for the Casson-Whitney number coming from the fundamental group of the knot complement. Finally, we compare it with the 1-handle stabilization number, another notion of "unknotting number" that has been in use for 2-knots.