GSCAGT 2018 Title and Abstract List

Talks marked with a dagger (†) indicate expository talks

Annular Khovanov homology (†)

Rostislav Akhmechet, University of Virginia

Khovanov homology is a powerful tool for studying links. In this talk, I will give a brief review of Khovanov homology from the viewpoint of Bar-Natan's cobordism category and describe the situation for links in a thickened annulus $S^1 \times I \times I$. There is a third grading on Khovanov homology in this case, which gives rise to variants of regular Khovanov homology such as sutured annular Khovanov homology. I will conclude with a brief survey of more recent applications of annular Khovanov homology.

Multiplicative structure of the Kauffman bracket skein algebra of a sphere with four holes

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Rhea Palak Bakshi, The George Washington University

In 1987, Józef Przytycki introduced skein modules as a way to extend the knot polynomials of the 1980s to knots and links in arbitrary 3-manifolds. Since their introduction, skein modules have become central to the theory of 3-manifolds. In 1997, Frohman and Gelca established a compact product-to-sum formula for the multiplication of curves in the Kauffman bracket skein algebra of $T^2 * [0, 1]$. We try to discover a similar formula for the multiplication of curves in $F_0 4 * [0, 1]$, the thickened sphere with four holes, and I will present some of our results to this end. This is joint work with Sujoy Mukherjee, Józef Przytycki, Marithania Silvero and Xiao Wang.

Hierarchically hyperbolic groups (†)

Daniel Berlyne, CUNY Graduate Center

Hyperbolic groups have been a focal point of geometric group theory since the idea was first introduced by Mikhail Gromov in 1987. Although most classes of groups in today's zeitgeist are nothyperbolic in general, they are still in some sense "close" to being hyperbolic; for example, mapping class groups, right-angled Artin groups, right-angled Coxeter groups, other cubical groups, and (most) 3-manifold groups. We would like a way to define this closeness in an explicit and useful way so that we can generalise tools that we already have, rather than study each of these classes of groups separately. One way of doing this is a recent construction of Jason Behrstock, Mark Hagen, and Alessandro Sisto in 2015: that of ahierarchically hyperbolic structure on a group.I will give a rough description of this structure, together with some examples.

Equivariant cohomology of spaces with a torus action and a compatible involution

Sergio Chaves, University of Western Ontario

Hamiltonian torus actions on symplectic manifolds have been widely studied in the last decades. M. Atiyah and T. Frankel proved that a symplectic manifold M and the fixed point subspace M^T have the same Betti sum; in particular, this implies that the equivariant cohomology $H_T^*(M,\mathbb{Q})$ is free as a module over the cohomology of the classifying space $H^*(BT,\mathbb{Q})$. It is natural to ask whether this situation still holds in a more general setting relying only on the topology of the space. The purpose of this talk is threefold; first, I will give a brief introduction on the generalities of Borel's equivariant cohomology. Secondly, I will exhibit M. Atiyah, T. Frankel and H. Duistermaat theorems on the equivariant cohomology of Hamiltonian torus actions on symplectic manifolds with a compatible involution. Finally, generalizations of these results on a merely topological setting will be discussed, including work of V. Guillemin, T. Holm, M. Franz and myself.

Thickness of $\mathrm{Out}(G)$

Saikat Das, Rutgers University-Newark

The notion of thickness was developed by Behrstock-Druţu-Mosher to determine non relative hyperbolicity for higher complexities of a large class of geometrically interesting groups, including $\mathcal{MCG}(S)$ and $\mathrm{Out}(F_n)$. In this talk we will inspect the thickness for $\mathrm{Out}(G)$, where G is a finite free product of finite groups.

On Fujita type conjectures

Yajnaseni Dutta, Northwestern University

In 1985, Fujita conjectured that for any smooth projective variety X of dimension n, $\omega_X \otimes L^\ell$ is globally generated for all $\ell \geq n+1$. While this can easily be seen in dimension 1, in dimensions ≥ 6 such an effective statement has been proven to be true only up-to a quadratic bound. Similar effective bounds have been proposed in the relative setting for the pushforwards of pluri-canonical bundles $\omega_X^{\otimes k}$. In this talk, I will present a recent work with Takumi Murayama, where in the relative setting we showed such global generations on an open set, and thereby proved an effective non-vanishing statement. I will also discuss some interesting positivity properties that seem to dictate such global generation statements.

Dualizability of bounded commutative BCK-algebras

Matt Evans, Binghamton University

Bounded commutative BCK-algebras are a variety of algebras (in the sense of universal algebra) that arise from a non-classical logic, and are generalizations of Boolean algebras. Following Stone's Duality for Boolean algebras then, it is natural to wonder if such a duality exists for bcBCK-algebras. As it turns out, the variety of bcBCK-algebras is not itself dualizable, but it has an infinite family of dualizable subvarieties. No knowledge of universal algebra will be assumed.

Degeneration of abelian differentials and period matrices

Xuntao Hu, Stony Brook University

The plumbing parameters give local coordinates near the boundary of the DeligneMumford compactification of the moduli spaces of curves. In this talk we introduce a new method to construct smooth abelian differentials near an arbitrary boundary strata. This method further allows us to compute the variational formula of the degenerating family of abelian differentials in terms of the plumbing parameters. We also give the variational formula for the degeneration of the period matrices, generalizing results of Yamada and Taniguchi. This is a collaborative work with Chaya Norton.

Every surface knot group is the group of a ribbon surface

Jason Joseph, University of Georgia

We give an elementary, constructive proof that every surface knot group is the group of an orientable ribbon surface. This allows us to define the ribbon genus of a connected surface as the minimum genus of any ribbon surface with the same group. For any 2-bridge knot K, we give a ribbon torus with the same group as any nontrivial twist spin of K; these are likely to achieve their ribbon genus, and in many cases this is known to be true. We conclude by investigating the existence of knotted spheres with arbitrarily high ribbon genus and the relationship between ribbon genus, deficiency, and triple point number.

Algebraic topology in neuroscience

Alex Kunin, Pennsylvania State University

Sensory neural networks encode stimuli via combinatorial patterns of spiking and silence, which correspond to intersection patterns of subsets of Euclidean space. These sets are often observed to be convex, which can lead to deep insights into the structure of stimulus representation and network architecture. This motivates an open problem, to determine the topological and geometric properties of the cover given its intersection patterns. I will give an outline of the problem and some recent results (including my own!). This talk should be accessible to any student who has taken an algebraic topology course.

Normal generators for mapping class groups are abundant

Justin Lanier, Georgia Institute of Technology

Under what conditions do a group element and all of its conjugates form a generating set for the ambient group? Such an element is called a normal generator. For mapping class groups of surfaces, we provide a number of simple criteria that ensure that a mapping class is a normal generator. We then apply these criteria to show that every nontrivial periodic mapping class that is not a hyperelliptic involution is a normal generator whenever genus is at least 3. We also show that every pseudo-Anosov mapping class with stretch factor less than $\sqrt{2}$ is a normal generator. Our pseudo-Anosov results answer a question of Darren Long from 1986. This is joint work with Dan Margalit.

Identifying “nice” convex neural codes

Caitlin Lienkaemper, Pennsylvania State University

Convex neural codes arise from the activity of neurons with convex receptive fields, such as hippocampal place cells. Mathematically, these correspond to the intersection patterns of convex open sets in Euclidean space. Here, we focus on codes which have a convex realization in general position, i.e. the code is stable under small perturbations of the ”receptive fields”. We identify two obstructions to a code being realizable with convex open sets in general position defined in terms of a certain cubical complex being connected and simply connected, and speculate about what the higher dimensional analogues to these obstructions might look like. Finally, we use these new tools to give a few surprising results on the maximal general position embedding dimensions of some simple codes.

First order properties of the free monoid

Laura Lopez Cruz, CUNY Graduate Center

We have studied model-theoretic questions about the free monoid $\mathbb{M}_X$ with finite basis X, namely the definability of finitely generated submonoids, homogeneity of the structure, and quantifier elimination of the theory. It was shown by W.V. Quine [**?**] that arithmetic is interpretable in the free monoid, and consequently its theory is undecidable and unstable. Moreover, the rank of the free monoid is definable, so free monoids of different ranks have different first-order theories. We have shown that $\mathbb{M}_X$ is bi-interpretable with the list superstructure of arithmetic, denoted as $S(\mathbb{N}, \mathbb{N})$. The theory of this structure has the same expressive power as the weak second order theory of $\mathbb{N}$, and it is known to be bi-interpretable with $\mathbb{N}$. Using this we proved that all submonoids of $\mathbb{M}_X$ with k generators are definable by the same formula, that is, for any $k \in \mathbb{N}$, there is a formula $\phi(x_1, ..., x_k, y)$ such that $\phi(g_1, ..., g_k, g)$ holds in $\mathbb{M}_X$ if and only if $g \in < g_1, ..., g_k >$. We have also shown that the theory of the free monoid does not admit quantifier elimination to any set Σ_n or Π_n, and that the structure is homogeneous. We also showed that the theory of the structure $S(\mathbb{M}_X, \mathbb{N})$ is bi-interpretable with $\mathbb{M}_X$. An interesting question is to find groups $\mathcal{G}$ that satisfy this, that is, $\mathcal{G}$ is bi-interpretable with $S(\mathcal{G}, \mathbb{N})$. This project is joint work with Olga Kharlampovich.

The Price twist via trisections

Maggie Miller, Princeton University

The Price twist is a surgery operation on RP^2s in 4-manifolds. In S^4, this operation yields a homotopy 4-sphere, and so is a tool in creating potential counterexamples to the smooth Poincaré conjecture. In this talk, I will show our new method of actually performing this surgery operation via the theory of trisections (to be explained in the talk). On the way, we will learn how to explicitly trisect the complement of any surface in a 4-manifold. This talk will be heavy on pictures and geometric thinking, and is joint with Seungwon Kim.

Upper density bounds for two-radius packings of disks in the plane

Ali Mohajer, University of Illinois at Chicago

Define the homogeneity of a packing of disks in the plane to be the infimum of the ratio of radii of disks in the packing. It has been known since 1953 (L. Fejes-Toth) that if the homogeneity of a packing is close enough to 1, the density of that packing cannot exceed $\frac{\pi}{\sqrt{12}}$, the upper bound on the density of a single-radius packing. "Close enough" was refined in 1963 by August Florian to mean a homogeneity in the interval (0.902, 1], and in 1969, Gerd Blind extended the left bound of this interval to approximately 0.742.
In 2003, sharp upper density bounds were established by Aladar Heppes for a handful of two-radius packings at homogeneities which admit arrangements wherein each disk is tangent to a ring of disks, each of which is tangent to its two cyclic neighbors. In this talk we will develop methods for establishing upper density bounds for saturated two-radius packings of disks when no such regularity exists, and discuss recent progress in establishing a bound sharper than the best one known for a specific ratio of radii.

Coordinates on Coulomb branches

Benedict Morrissey, University of Pennsylvania

In certain quantum field theories there is an integrable system, points of which describe lowest energy states. We will describe a set of co-ordinate charts on this integrable system, generalizing earlier work of Gaiotto–Moore–Neitzke. Given time the relation to the wall crossing of Kontsevich and Soibelman will be discussed. No knowledge of quantum field theory or integrable systems will be assumed.

Artin groups and an associated CAT(0) cube complex

Rose Morris-Wright, Brandeis University

Artin groups form a large class of groups including braid groups, free groups, and free abelian groups. They are related to Coxeter groups and have nice geometric properties. However many questions about the properties of Artin groups remain unanswered. In this talk, I will introduce the clique-cube complex, a CAT(0) cube complex constructed from a given Artin group. I will discuss some of the properties of this cube complex, as well as how it can be used to show that a large class of Artin groups have trivial center.

Taut sutured handlebodies as twisted homology products

Margaret Nichols, University of Chicago

A basic problem in the study of 3-manifolds is to determine when geometric objects, such as Seifert surfaces or surfaces in a fixed homology class, are of ‘minimal complexity. We are interested in this question in the setting of sutured manifolds, which generalize knot complements equipped with a fixed Seifert surface. In this setting, minimal complexity is called ‘tautness.

One method for certifying that a sutured manifold is taut is to show that it is homologically simple - a so-called ‘rational homology product. Most sutured manifolds do not have this form, but do always take the more general form of a ‘twisted homology product’, which incorporates the added algebraic information of a representation of the fundamental group. The question then becomes, how complicated of a representation is needed to realize a given sutured manifold as such?

We explore some classes of relatively simple sutured manifolds, and see one class is always a rational homology product, but that the next natural class contains examples which require twisting. We also find examples that require twisting by a representation which cannot be ‘too simple.

Do these pants still fit? (†)

Adriano Pacifico, University of Toronto

In his groundbreaking preprint from 1986, Thurston describes a new geometry on Teichmüller space by introducing what is now known as the Thurston metric. Aside from not actually being a metric, the Thurston metric measures the largest amount by which you have to stretch curves from one surface to the other. In this talk, I will describe a map from Teichmüller space to the space of measured foliations transverse to a fixed geodesic lamination. By scaling the measure in a manner dependent on time, we obtain a path in Teichmüller space, called the *stretch path*, which happens to be a geodesic in the Thurston metric. In fact, Thurston used stretch paths to show that Teichmüller space with the Thurston metric is a geodesic metric space.

Toric symplectic and origami manifolds

Ian Pendleton, Cornell University

Toric symplectic manifolds are spaces of particular interest to symplectic geometers because their complicated geometry is completely classified by special polytopes that are easy to work with and visualize. I’ll introduce symplectic and toric symplectic manifolds, and then talk about a generalization of toric symplectic manifolds called toric origami manifolds. Toric origami manifolds are classified by “origami templates,” which in 2 dimensions resemble the folded paper art for which they are named.

Studying weakly superslice links with virtual knot theory

Puttipong Pongtanapaisan, University of Iowa

A 2-component link L in S^3 is called *weakly superslice* if it bounds a smoothly embedded annulus in B^4 whose double along L produces an unknotted torus in S^4. After a brief review of virtual knot theory, I will discuss how one can determine whether a 2-component link is weakly superslice by studying a virtual knot invariant called the Wirtinger number.

Autoequivalences of twisted K3 surfaces

Emanuel Reinecke, University of Michigan

The autoequivalences of a variety are natural generalizations of its automorphisms. For complex, projective K3 surfaces, Bridgeland conjectured an alluring description of the group of autoequivalences. In this talk, we first review some background on K3 surfaces and derived categories and give a precise definition of the notion of autoequivalences. Then, we explain how passing to the larger class of twisted K3 surfaces can be used to prove one part of Bridgeland's conjecture.

Determining groups by their boundaries

Jacob Russell, CUNY Graduate Center

Introduced by Cordes, the Morse boundary of a proper geodesic metric space generalizes the fruitful Gromov boundary of hyperbolic spaces. The Morse boundary captures the asymptotics of the "hyperbolic-like" directions in the space and agrees with the Gromov boundary when the space is hyperbolic. The salient property of both the Morse and Gromov boundaries is that any quasi-isometry between spaces will induce a homeomorphism of the boundaries. While the converse is not true in general, a classic result of Paulin's says that *quasi-möbius* homeomorphisms between Gromov boundaries do induce quasi-isometires of the underlying spaces. Recently, Charney and Murray established a Morse boundary version of Paulin's theorem for CAT(0) groups and in this talk we will present an extension of the work of Charney and Murray to the class of hierarchically hyperbolic spaces, a class of metric spaces including mapping class groups, many 3-manifold groups and right angled Artin groups. This is joint work with Sarah Mousley.

Virtual specialness of right-angled 3-manifolds (†)

Alexander Stas, CUNY Graduate Center

A right-angled 3-manifold is a complete, finite volume hyperbolic 3-manifold which admits a decomposition into a finite collection of right-angled ideal hyperbolic polyhedra glued together via face pairing isometries. In this talk, I will outline Chesebro-Deblois-Wilton's proof of the virtual specialness of right-angled 3-manifolds. I will show that such manifolds have a natural deformation retract onto a virtually special square complex. By a result of Agol, this implies that right-angled 3-manifolds are virtually fibered.

Topology of hybrid analytifications

Matt Stevenson, University of Michigan

Given a variety over a complete normed field, one can construct its Berkovich analytification; in fact, a similar process works over more general bases, e.g. certain Banach rings. Such analytifications are called hybrid spaces, as the fibres of the structure map are Berkovich analytifications over different normed fields. In this way, one can for example interpolate between the usual analytification of a variety over the complex numbers and the trivially-valued Berkovich analytification of the variety. We will discuss some recent work (joint with T. Lemanissier) on the topology of these hybrid spaces.

Cubulable quotients of free products

Ben Stucky, University of Oklahoma

In 2015, Martin and Steenbock proved that a small cancellation quotient of a free product acts properly and cocompactly on a CAT(0) cube complex if the factors do, thus verifying a powerful non-positive curvature condition for these groups. To do this, they construct a nicely-behaved wallspace structure in the universal cover which builds upon the existing wallspace structures of the factors and invoke the Sageev construction. Shortly thereafter, Jankiewicz and Wise placed a special case of their results in the context of Wises cubical small cancellation theory. I will describe work in progress seeking to cubulate some quotients of free products which are not necessarily small cancellation.

Masur's criterion does not hold in Thurston metric

Ivan Telpukhovskiy, University of Toronto

We provide a counterexample for Masur's criterion in the setting of Teichmüller space with Thurston metric. For that, we first construct a minimal non-uniquely ergodic lamination λ on a seven-punctured sphere from a sequence of curves with bounded twisting property. Then we show that the geodesic in the corresponding Teichmüller space that converges to λ, stays in the thick part for the whole time.

Strong contractibility of geodesics in the mapping class group

Yvon Verberne, University of Toronto

A geodesic is strongly contracting if its nearest point projection takes disjoint balls from the geodesic to sets of bounded diameter, where the bound is independent of the ball. In joint work with Kasra Rafi, we show that the axis of a pseudo-Anosov homeomorphism in the mapping class group may not have the strong contractibility property. In particular, we show that it is possible to choose an appropriate generating set for the mapping class group of the five-times punctured sphere so that there exists a pseudo-Anosov homeomorphism ϕ, a sequence of points x_k, and a sequence of radii R_k so that the ball $B_{R_k}(x_k)$ is disjoint from the axis of ϕ, but the closest point projection of $B_{R_k}(x_k)$ to the axis is at least $c\log(R_k)$. Along the way, we show that it is, in fact, possible to construct explicit geodesics in the mapping class group.

Hearing the limiting shape of a hypersurface configuration

Robert Walker, University of Michigan

There is a niche body of work on limiting shapes, i.e., asymptotic Newton polyhedra, of symbolic generic initial systems considered for polynomial rings in characteristic zero (e.g., by my academic sister Sarah Mayes-Tang, and separately by Dumnicki, Szemberg, Szpond, and Tutaj-Gasińska, a quartet of Polish mathematicians). In particular, in one joint paper the latter four authors compute the limiting shape for ideals defining zero-dimensional star configurations in projective space–star configurations turn out to be a steady source of a lot of interesting "ALGECOM" phenomenology. In this talk, we discuss work-in-progress to generalize their computation to the case of ideals defining zero-dimensional configurations in projective space determined by hypersurfaces of a common fixed degree. Along the way, we draw connections to a 2015 investigation (published in Transactions of the AMS in 2017) of select homological and asymptotic properties of hypersurface and matroidal configurations by Geramita, Harbourne, Migliore, and Nagel. I'll aim to close the talk by indicating how we might see–or "hear"–select asymptotic numerical invariants in the limiting shape: Waldschmidt constants, asymptotic Castelnuovo-Mumford regularity, and resurgences for homogeneous polynomial ideals.

An introduction to the Chabauty topology (†)

Sangsan Warakkagun, Boston College

The space of closed subgroups of a Lie group could be endowed with a natural compact topology, called the Chabauty topology. It proves useful in the study of geometry. For example, by taking the Lie group to be the isometry group of the hyperbolic space, we can define a natural notion of the limit of a sequence of hyperbolic manifolds.

In this talk, besides introducing the Chabauty topology, I will provide examples of the Chabauty spaces of some Lie groups.

Hodge theory and combinatorics (†)

Weihong Xu, Rutgers University

In 2015, Adiprasito, Huh, and Katz gave an algebraic geometry motivated proof of some conjectures in combinatorics. The conjectures are about a common phenomenon called log-concavity. A sequence $a_0, \ldots, a_d$ is log-concave if $a_i^2 \geq a_{i-1}a_{i+1}$ for all i. The binomial coefficients $\binom{n}{k}$ for n fixed and $k = 0, \ldots, n$, for instance, are log-concave. The conjectures are stated using the concept of a matroid, which can be viewed as combinatorial abstractions of graphs and the notion of linear independence of vectors. The log-concavity of many sequences, such as the coefficients $\chi_G(t)$ of the chromatic polynomial ($\chi_G(t)$ is the number of proper colorings of a finite graph G using t colors) follow as special cases from the conjectures. I will attempt to introduce matroids as well as some constructions and results in the paper by Adiprasito, Huh, and Katz, including the matroidal Chow rings, Poincare Duality for matroids, hard Lefschetz property, Hodge-Riemann Relations, and talk about some related notions and results in algebraic geometry. For an interesting story about Huh and the proof, see `https://www.quantamagazine.org/a-path-less-taken-to-the-peak-of-the-math-world-20170627/`. This page (`http://math.columbia.edu/~rcheng/F2017.html`) of Raymond Cheng (who has been helping me understand the work) lists some useful references.

Application of toric systems to several conjectures on surfaces

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Shizhuo Zhang, Indiana University

After recalling so called toric systems defined by Hille-Perling, I will apply this technical tool to attack several problems on surfaces: Orlov's folklore conjecture on rationality of surfaces, Daniel Chan's conjecture on classifications of 2-hereditary tilting bundles, the coincidence of several notions of cyclic strong exceptional collections of line bundles arising from different areas in mathematics. Moreover, I will use it to improve a result by Duo Li on categorical Kobayshi theorem and use it to understand Brill-Noether problems for surfaces.
