GTA: Philadelphia 2023 Title and Abstract List

Keynote Talks

Knots, groups, and 3-manifolds

Jennifer Hom, Georgia Institute of Technology

Knots in the 3-sphere, under the operation of connected, form a monoid. However, lacking inverses, this fails to be a group. We remedy this by considering knots modulo an equivalence relation called concordance, to obtain the knot concordance group. We discuss various properties of this group. We will also discuss the closely related 3-dimensional homology cobordism group.

Geometric structures associated to higher-rank Anosov representations

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Sara Maloni, University of Virginia

The Teichmüller space of a surface S is the space of marked hyperbolic structure on S. By considering the holonomy representation of such structures, it can also be seen as a connected component of representations from the fundamental group of S into $\mathrm{PSL}(2, \mathbb{R})$ consisting entirely of discrete and faithful representations. Generalizing this point of view, Higher Teichmüller Theory studies connected components of representations from the fundamental group of S into more general semisimple Lie groups like $\mathrm{PSL}(2, \mathbb{R})$ which consist entirely of discrete and faithful representations.

In this talk we will give a survey of some aspects of Higher Teichmüller Theory, and will make links with the recent powerful notion of Anosov representations. We will describe joint work with Daniele Alessandrini, Nicolas Tholozan and Anna Wienhard where we describe how most of these representations correspond to deformation of geometric structures on smooth fiber bundles over S.

The Mighty Laplacian and Symmetry: Generic Properties of Laplace Eigenfunctions

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Craig Sutton, Dartmouth College

The study of the Laplace operator and its eigenfunctions has been of interest to mathematicians and physicists for centuries. For instance, in the late eighteenth and early nineteenth century, Chladni intrigued audiences with experiments exhibiting intricate patterns (i.e., nodal sets) formed by sand sprinkled across a vibrating plate. And, in quantum mechanics, Laplace eigenfunctions are interpreted as probability density functions associated to the position of a free particle. Inspired by work of Karen Uhlenbeck, we will explore generic properties of Laplace eigenfunctions focusing on recent results involving spaces with non-trivial symmetry groups.
This is joint work with Donato Cianci (GEICO), Chris Judge (Indiana) and Samuel Lin (Oklahoma).

Effective methods in inverse Galois theory

John Voight, Dartmouth College

Is every finite group a Galois group over the rationals? Unfortunately, we don't know in general, even though much ink has been spilled trying to answer this question, from many perspectives! In this talk, we spill a bit more: we report on joint work concerning effective methods from topology and geometry to realize Galois groups (with explicit or even nice polynomials, in practice or at least in principle).

Student Talks

Talks marked with a dagger (†) indicate expository talks.

Minimizing Denominators and Geometric Generalizations

Albert Artiles, University of Washington

We provide a geometric interpretation for a normalized version of the minimal denominator,

$$q_{\min}(x,\delta) = \min \left\{ q \in \mathbb{N} : \text{ there exists } p \in \mathbb{Z} \text{ such that } \frac{p}{q} \in (x-\delta, x+\delta) \right\},$$

function introduced by Chen and Haynes. We use this interpretation to compute the limiting distribution of $\delta^{\frac{1}{2}} q_{\min}(x,\delta)$ as a function of x, and give generalizations to the idea of minimal denominators to general unimodular lattices, linear forms, and translation surfaces. The key idea is to turn this circle of problems into equidistribution problems for translates of unipotent orbits of a Lie group action on an appropriate moduli space.

Geometric Representatives for Homology

Spencer Cattalani, Stony Brook University

Given a homology class on a manifold, how geometrically nice a representative can one find? This is a classic question in topology with applications to symplectic topology and gauge theory. In this talk, I will survey results on this question, including recent work on non-compact manifolds.

A variant of Mom technology for hyperbolic multi-cusped manifolds

Joye Chen, Princeton University

In the 2000s, Gabai, Meyerhoff, and Milley introduced Mom technology for complete hyperbolic manifolds and applied it to successfully enumerate all 1-cusped manifolds with volume ≤ 2.848. We introduce a variant of Mom technology, apply it to multi-cusped hyperbolic 3-manifolds and present partial results towards finding simple internal generalized Mom structures in hyperbolic 3-cusped 3-manifolds with volume ≤ 5.3335.

We also outline a program to prove that the manifold s776 is the unique minimal volume hyperbolic three-cusped 3-manifold and describe connections to the Hyperbolic Complexity Conjecture of Thurston, Weeks, Matveev-Fomenko, and others. This talk is based on the undergraduate thesis work of the speaker advised by Prof. David Gabai.

Fox Coloring Group and Alexander-Burau-Fox Module of Wheel Graphs

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Anthony Christiana, George Washington University

For a plane graph G, we construct the link diagram $D(G)$ and compute the reduced Fox Coloring Group $Col^{\text{red}}(D(G))$ of the diagram of the wheel graph of n spokes, $D(W_n)$. This diagram can be presented as the closure of the braid $(\sigma_1\sigma_2^{-1})^n$, which are sometimes called Turkhead links. We show that $Col^{\text{red}}(D(W_n)) = \mathbb{Z}_{F_n-1+F_{n+1}} \oplus \mathbb{Z}_{F_{n-1}+F_{n+1}}$ when n is odd and $\mathbb{Z}_{5F_n} \oplus \mathbb{Z}_{F_n}$ when n is even, where F_i are Fibonacci numbers. We generalize this result to compute the Alexander-Burau-Fox matrix A_n over the ring $\mathbb{Z}[t^{\pm 1}]$ and show that $A_n = -g_n \begin{bmatrix} g_{n+1} + t^{-1}g_{n-1} & -t^{-1}g_{n-1} \\ -tg_{n+1} & g_{n+1} + tg_{n-1} \end{bmatrix}$ where $g_n = S_{k-1}$ for $n = 2k$ and $g_n = S_{k-1} + S_k$ for $n = 2k + 1$ and S_k is the Chebyshev polynomial of the second kind.

On On Cohomological Dimension of Group Homomorphisms

Aditya De Saha, University of Florida

Classically, cohomological dimension of a group G is the maximal number n such that the n-th cohomology group of G with some coefficients is nontrivial. Recently there was introduced a similar notion of cohomological dimension for any group homomorphism $\phi : G \to H$. I will try to outline some basic results, and properties of this $cd(\phi)$. Then two main results will be shown, $cd(\phi \times \phi) = 2cd(\phi)$, and $hd(\phi) = cd(\phi)$. (For geometrically finite groups G,H). Time permitting I will try to describe some open problems, and some differences with cd of groups . This is joint work with Alexander Dranishnikov.

Geometry of Selberg's bisectors in the symmetric space $SL(n, \mathbb{R})/SO(n, \mathbb{R})$

Yukun Du, University of California, Davis

We discussed some properties of a family of symmetric spaces, namely $SL(n, \mathbb{R})/SO(n, \mathbb{R})$, where we replace the Riemannian metric on $\mathcal{P}(n)$ with a premetric suggested by Selberg. These include intersecting criteria of Selberg's bisectors, the shape of Dirichlet-Selberg domains, and angle-like functions between Selberg's bisectors. These properties are generalizations of properties on the hyperbolic spaces $\mathbf{H}^n$ related to Poincaré's fundamental polyhedron theorem.

Periodic Points of Veech Surfaces

Sam Freedman, Brown University

A Veech surface is a translation surface whose (affine) automorphism group is as large as possible. While a generic point of a Veech surface equidistributes under the action of its automorphism group, there is an exceptional finite set of points with finite orbits. These periodic points appear throughout Teichmüller dynamics, such as in counting holomorphic sections of families of Riemann surfaces and in blocking problems on billiard tables. In this talk we first describe joint work with Zawad Chowdhury, Samuel Everett and Destine Lee that gives an algorithm that computes the set of periodic points for a given Veech surface. We then describe work that classifies the periodic points of Prym eigenforms, an infinite family of Veech surfaces in genera 2, 3 and 4.

Special Cube Complexes embed in RAAGs (†)

Pratit Goswami, University of Oklahoma

Special cube complexes are introduced as cube complexes whose immersed hyperplanes behave in an organized way and avoid various forms of self-intersections. In this talk, we will see Haglund and Wise's characterization of special cube complexes – how their fundamental groups embed in right angled Artin groups (RAAGs) because of a locally isometric immersion to the cube complex of a naturally associated RAAG. I will then talk about the canonical completion and retraction which allow us to prove: Geometric subgroups (represented by a locally isometric immersion of cube complexes) of special groups are separable.

Flattened Stirling Permutations and Type B Set Partitions

Kimberly Harry, University of Wisconsin - Milwaukee

Recall that a Stirling permutation is a permutation on the multiset $\{1, 1, 2, 2, 3, 3, \dots, n, n\}$ such that any numbers appearing between repeated values of i must be greater than i. We call a Stirling permutation "flattened" if the leading terms of maximal chains of ascents (called runs) are in weakly increasing order. Our main result establishes a bijection between flattened Stirling permutations and type B set partitions of $\{0, \pm 1, \pm 2, \dots, \pm n-1\}$. This readily implies that flattened Stirling permutations of order n are enumerated by the Dowling numbers.

Volumes of Jacobians and Pryms (†)

Junaid Hasan, University of Washington

Kirchhoff's matrix tree theorem is a classical result says that the number of spanning trees of a graph is the determinant of the reduced laplacian matrix of the graph. Infact, this number can be reinterpreted as the order (or size) or a group known as Jacobian. Furthermore we can interpret the size of the jacobian as a volume computation, thereby giving a geometric meaning to this celebrated theorem. Infact, we can investigate morphisms between graphs, in particular double covers (known as prym double covers) and provide geometric interpretation (volume of a certain polyhedral set) to generalizations of matrix tree theorems. This is based on An, Baker, Kuperberg, Shokrieh [1304.4259] and Len, Zakharov [2012.15235] and further ongoing work.

The Hot Spots Conjecture for Hyperbolic Triangles

Lawford Hatcher, Indiana University Bloomington

The hot spots conjecture of J. Rauch states that the extrema of a second Neumann eigenfunction of the Laplace-Beltrami operator on a bounded domain occur only on the boundary. Judge and Mondal proved in 2021 that the conjecture holds for all Euclidean triangles by analyzing zero-level sets of various derivatives of the eigenfunctions and the behavior of the eigenfunctions at the vertices. We will demonstrate that these methods apply with minor modifications to prove the conjecture in the case of obtuse hyperbolic triangles, and, time permitting, we will discuss the obstructions to using these methods in the acute case.

Lefschetz fixed point theorem on stratified pseudomanifolds

Gayana Jayasinghe, University of Illinois Urbana Champaign

The Atiyah-Bott-Lefschetz fixed point theorem was one of the major breakthroughs in equivariant index theory, leading to equivariant localization which is now key in many areas of mathematics and physics.
I will present a generalization of the L^2-Atiyah-Bott-Lefschetz fixed point theorem for elliptic complexes on stratified pseudo manifolds, focusing on the cases of the Dolbeault and spin Dirac complexes on singular algebraic varieties with explicit computations in a range of examples.
I will compare and contrast this with old equivariant Riemann-Roch formulas and Todd classes for singular algebraic varieties, including those of Baum-Fulton-MacPherson-Quart, as well as in intersection homology by Goresky and MacPherson and discuss new formulations and perspectives on some of them.

Computing Cohomology of Singular K3 Surfaces

Steppan Konoplev, University of Delaware

The cohomology of smooth K3 surfaces is well-known, having been computed completely by Kodaira in 1960. We extend his work by computing the cohomology of singular K3 surfaces which appear in Iano-Fletcher's paper "Working with Weighted Complete Intersections" by blowing them up at points several times until they become smooth. Specifically, all K3 surfaces with A_n type singularities are handled. We prove a theorem allowing us to compute the cohomology of a surface from the cohomology of its blowup, then apply it to undo each blowup to arrive at the cohomology of the original surface.

The LS-category of group homomorphism

Nursultan Kuanyshov, University of Florida

The Lusternik-Schnirelmann category (LS-category) is an important homotopy invariant. On 50s, Eilenberg and Ganea proved that the LS-category of a discrete group equals its cohomological dimension, $cat(\Gamma) = cd(\Gamma)$. Jamie Scott conjectured that similar inequality for group homomorphism, $cat(\phi) = cd(\phi)$ where $\phi : \Gamma \to \Lambda$. In this talk, we give the recent development in this direction.

Homological Representations of Low Genus Mapping Class Groups

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Trent Lucas, Brown University

In this talk, we discuss homeomorphisms of surfaces and their action on homology. That is, any homeomorphism of a surface S induces a map on homology, yielding a representation $\mathrm{Homeo}(S) \to \mathrm{GL}(H_1(S))$. We first discuss the classical theorem that the image of this representation is the symplectic group $\mathrm{Sp}(2g, \mathbb{Z})$. Then, we consider an equivariant refinement: if G is a finite group acting on S, our representation restricts to a map on centralizers $\mathrm{Homeo}(S)^G \to \mathrm{GL}(H_1(S))^G$. We discuss our recent result that when S has small genus, the image of this refined representation is always an arithmetic group.

Bounds on the Cross-Cap Number (non-orientable Genus) of links

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Rob McConkey, Michigan State University

The cross-cap number of a link is an invariant which considers the non-orientable spanning surfaces of the link, similar to how the genus of a link depends on the orientable spanning surfaces. In 2014 Kalfagianni and Lee found linear bounds for the cross-cap number of alternating links in relation to the coefficients of the Jones Polynomial. But what happens when we begin to look beyond alternating links? We consider a couple of families of links where such linear bounds cannot be found. Then talk about a family where we can find linear bounds for the cross cap number with respect to the twist number.

Hierarchically Hyperbolic Groups (†)

Zhihao Mu, The Graduate Center, CUNY

Hierarchically hyperbolic spaces/groups(HHS/HHG), introduced by Behrstock, Hagen and Sisto who were motivated by the successful Masur-Minsky's machinery for studying mapping class groups, become a useful tool to study many groups in geometric group theory, for example mapping class groups of finite type surfaces, right-angled Artin groups and right-angled Coxeter groups. In this talk, I will introduce some important and useful properties of HHG and explain the relation between HHS and CAT(0) cube complexes.

Esquisses des quelques programmes

James Myer, The Graduate Center, CUNY

I'll sketch two programs regarding progress toward the problem of resolution of singularities.
One perspective is that of a frog (in the words of Freeman Dyson this is not meant with any offense: I happen to adore frogs): given a hyperelliptic curve over a (insert pleasant adjectives here) field whose ring of integers is of mixed characteristic (0, 2), produce a regular model. This project is advised by Andrew Obus.
Another, more akin to a bird: establish topological obstructions to the existence of a resolution of the singularities of a variety (over a field of any characteristic, e.g. positive characteristic). In fact, the existence of alterations suggests there are no "étale obstructions". This project is advised by Dennis Sullivan.

Boundary behaviour of geodesics in asymptotically symmetric spaces

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Achinta Nandi, Oklahoma State University

We study the boundary behaviour of geodesics in asymptotically hyperbolic (AH) and asymptotically complex hyperbolic (ACH) spaces. In the AH case, we simply recover the earlier results of Mazzeo but we give a new treatment of this case using a canonical symplectic structure on the cotangent bundle of Mazzeo's 0-cotangent bundle to describe the geodesic flow as a cotangent flow that is (sufficiently) regular up to the boundary. This approach gives a model for the more complicated situation of asymptotically complex hyperbolic spaces, which we handle in an analogous fashion by introducing and using a canonical symplectic structure on Epstein-Melrose-Mendoza's Θ-cotangent bundle.

Successive minima of lattices of cusp forms

Souparna Purohit, University of Pennsylvania

Given a lattice L in a Euclidean space V, its ith successive minima is the smallest real number λ such that L contains i linearly independent elements of norm at most λ. For instance, the first successive minima is the length of the shortest non-zero lattice vector. Calculating the successive minima for a given lattice is hard in general - in fact, there are post-quantum crypto-systems based on the difficulty of calculating these numbers.

Now given an arithmetic variety $\mathscr{X}$ and a Hermitian line bundle $\mathscr{L}$ on $\mathscr{X}$, we are interested in the successive minima of the lattice $\mathscr{M}_k := H^0(\mathscr{X}, \mathscr{L}^{\otimes k})$ in the Euclidean space $\mathscr{M}_k \otimes_{\mathbb{Z}} \mathbb{R}$ endowed with the sup (or L^2) norm coming from the metric on $\mathscr{L}$. When the metric is smooth, or even continuous, work of Huayi Chen describes the distribution of the successive minima of $\mathscr{M}_k$ in the limit as k tends to ∞.

We study the above problem when the metric on $\mathscr{L}$ is *logarithmically singular*. In the special case when $\mathscr{X}$ is a proper regular model of the modular curve associated with a finite-index subgroup $\Gamma \subseteq \mathrm{PSL}_2(\mathbb{Z})$, and $\mathscr{L}^{\otimes k}$ is an integral model of the line bundle of Γ-modular forms of weight $12k$ endowed with the Petersson metric, we describe the distribution of the successive minima of the sub-lattice of $\mathscr{M}_k$ comprising of the cusp forms (with respect to the Petersson norm). We highlight in particular the differences between our conclusion and that of Chen's.

Iwasawa λ invariant and Massey product

Peikai Qi, Michigan State University

How does the class group of number field change in field extensions? This question is too wild to have a uniform answer, but there are some situations where partial answers are known. I will compare two such situations. First, in Iwasawa theory, instead of considering a single field extension, one considers a tower of fields and estimates the size of the class groups in the tower in terms of some invariants called lambda and mu. Second, in a paper of Lam-Liu-Sharifi-Wake-Wang, they relate the relative size of Iwasawa modules to values of a "generalized Bockstein map", and further relate these values to Massey products in Galois cohomology in some situations. I will compare these two approaches to give description of the cyclotomic Iwasawa lambda-invariant of some imaginary quadratic fields in terms of Massey products.

Einstein's Field Equations Vs Constraint Equations (†)

Siddiqur Rahman, Oklahoma State University

Einstein's gravitational field equations(EFEs) can be considered the most celebrated (arguably) system of PDEs of the last century. Combing Guass and Codazzi equations with Einstein's field equations, one obtains the four Einstein's constraint equations that must satisfy the initial data for the solution of EFEs. In this talk, we show the geometric origin of Einstein's constraint equations and their role to construct solutions to EFEs.

The $\mathrm{SL}_2\,\mathbb{C}$ Character Variety of a Hyperbolic 3-Manifold (†)

Iris Rosenblum-Sellers, University of California, Berkeley

In 1983, Culler and Shalen used the character variety of representations of the fundamental group of a hyperbolic 3-manifold with boundary to produce essential surfaces in the manifold. This technique was employed to prove the weak Neuwirth conjecture on separating essential surfaces in nontrivial knot complements, helped to prove the Smith conjecture on torsion diffeomorphisms of S^3, and was instrumental in proving the cyclic surgery theorem, key to the result that knots are determined by their complements. I will discuss the key ideas and techniques, and give some indication of what's happened over the last 40 years.

The non-existence of geometric connected sums of conformally flat Lorentzian manifolds

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Geoffrey Sangston, University of Maryland

Ravindra S. Kulkarni exhibited a construction of a geometric connected sum of two conformally flat Riemannian manifolds, which itself admits a conformally flat Riemannian structure that is induced by the structures on the two factors. Charles Frances later exhibited a similar surgery construction which combines two conformally flat Lorentzian manifolds by excising certain solid torus neighborhoods of lightlike geodesics and gluing the resulting boundaries. He then asks if the connected sum of two conformally flat Lorentzian manifolds can be endowed with a conformally flat Lorentzian structure. In the case of closed even dimensional manifolds, a negative answer to this question can be given by considerations involving the Euler characteristic. In the odd dimensional case, we also give a negative answer to a natural interpretation of this question. This follows by assuming the existence of a geometric connected sum of conformally flat Lorentzian manifolds, and then using this assumption to construct a sphere with a conformally flat Lorentzian metric, which contradicts a fact following from the basic theory of developing maps that spheres cannot carry conformally flat Lorentzian metrics. Time permitting, generalizations of this construction will be discussed.

Some conditions for non-trivial Z_p-extensions of imaginary quadratic fields

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Christopher Stokes, Arizona State University

Iwasawa theory is a branch of number theory that studies the behavior of certain objects associated to a

Z_p-extension. In this talk I will focus on the cyclotomic Z_p-extensions of imaginary quadratic fields for varying primes p, and will give some surprising conditions for when the corresponding lambda-invariants are greater than 1.

The Min-Oo Conjecture, Toponogov's Theorem, and Almost Rigidity Problems

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Hunter Stufflebeam, University of Pennsylvania

A rigidity theorem in geometry takes in a geometric object and, provided certain measurements on the object satisfy the right hypotheses, identifies the object as a known space. Given such a result, it can be natural to ask if the hypotheses are *close* to being satisfied implies that the geometric object in hand is somehow *close* to a known space. In this talk, Ill describe a family of famous rigidity theorems, and some recent efforts towards proving such stable versions.

Compactifying rank-one Weyl-parallel manifolds

Ivo Terek, The Ohio State University

The local types of essentially conformally symmetric manifolds (i.e., pseudo-Riemannian manifolds with parallel Weyl tensor which are not locally symmetric or conformally flat; in short, ECS manifolds) have been fully described by Derdzinski and Roter in 2009. They are distinguished by the rank, always equal to 1 or 2, of a certain null parallel distribution $\mathcal{D}$ associated with the Weyl tensor. Examples of compact rank-one ECS manifolds were only known to exist in dimensions of the form $n = 3k + 2$, starting from 5, but their construction raised more questions than answers. We present new examples in all dimensions starting from $n = 5$ via quotients of a certain "pp-wave" manifold. Such "pp-wave" manifolds constitute the universal coverings of a "generic" class of compact rank-one ECS . Topological features of the constructed examples are also not accidental: we show that outside the locally homogeneous case (and up to a double isometric covering), compact rank-one ECS manifolds must be total spaces of fibre bundles over the circle. This is joint work with Andrzej Derdzinski.

On Almost Strong Approximation in Algebraic Tori

Wojciech Tralle, University of Virginia

Let K be a global field and let S be a subset of valuations of K. Denote by $AF_K(S)$ the ring of S-adeles. We say that an algebraic group G defined over K has strong approximation property with respect to S if the diagonal embedding of the K-points of G into the $AF_K(S)$-adelic points of G, has dense image. Strong approximation has been extensively studied since M. Eichler and M. Kneser. It is known that algebraic tori cannot have strong approximation with respect to any finite set of places. We establish a version of strong approximation for tori, which we call almost strong approximation for certain infinite sets S. More precisely, we show that if S contains some generalized arithmetic progression and all archimedean valuations, minus a set of zero Dirichlet density then for any torus T defined over K, the quotient group of S-adelic points of T by the closure of K-points of T is finite. Furthermore, we prove that the order of this group is bounded by a function depending only on the dimension of T and the degree of the field extension defining the generalized arithmetic progression. This theorem also has an application to the Congruence Subgroup Problem.

Relatively Hyperbolic and Acylindrically Hyperbolic Dehn Fillings (†)

Ping Wan, University of Illinois, Chicago

We will start with an introduction to Dehn Surgery on 3-manifolds. We'll then delve into its group theoretic analogue, relatively hyperbolic Dehn Fillings. We'll explore how it has been used to prove the virtual Haken conjecture. Finally, we'll end with a brief discussion of its generalization to acylindrically hyperbolic groups.

Non-existence of Hölder embeddings into the Heisenberg Group (†)

Yizhou Zeng, University of Pittsburgh

The Heisenberg group, $\mathbb{H}^n$, is a sub-Riemannian manifold equipped with the Carnot-Caratheodory metric d_c. It is homeomorphic to $\mathbb{R}^{2n+1}$ but it has Hausdorff dimension of $2n+2$. It is known that given any $\alpha-$Hölder map from a 3-dimensional unit ball to $\mathbb{H}^1$, the image has Haudorff dimension at most $3/\alpha$. Hence for $\alpha > 3/4$, the image of 3-dimensional unit ball in $\mathbb{H}^1$ under any $\alpha-$Hölder map will not contain a metric ball. Gromov conjectured that if $f : \mathbb{R}^2 \to \mathbb{H}^1$ is an $\alpha-$Hölder embedding, then $\alpha \leq 1/2$.