Introduction to the Geometry of Sets and Measures

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Abstract

The familiar Lebesgue measure in Euclidean space gives a good way to measure the "length" of sets in the line $\mathbf{R}^1$, the "area" of sets in the plane $\mathbf{R}^2$, the "volume" of sets in space $\mathbf{R}^3$, and so on. But suppose, for example, that you want to measure the "length" of sets in $\mathbf{R}^n$ for $n > 1$. How should you do it? Attempts to answer this question led to the development of the branch of mathematics that is now called Geometric Measure Theory (GMT). In this course, I will give an overview of the subject, aimed at graduate students and mathematicians who are interested in analysis, broadly defined. The first half of the course (Lecture 1 and first half of Lecture 2) will introduce tools and techniques from GMT, to which I believe that every modern analyst should be exposed. The second half of the course (second half of Lecture 2 and Lecture 3) will outline some more specialized topics, which I feel illustrate the depth, beauty, and mystery of the geometry of sets and measures.

Lecture 1: covering theorems for doubling and Radon measures, differentiation of measures

Lecture 2: Hausdorff measures, Hausdorff densities, rectifiable and purely unrectifiable sets

Lecture 3: tangent measures, Preiss' theorem and uniform measures