
The Origin and Early Impact of the Moore Method

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1. INTRODUCTION. The Moore Method is the best known—and arguably the most successful—way to train students to become creative research mathematicians. Here the term Moore Method (as capitalized) will refer only to the method as it was used by R. L. Moore—one of the towering figures of mathematics in America [65]—and not by his students or their descendants. Figure 1 shows the octogenarian Moore in action. Mathematicians of a certain age know the main contours of the Moore Method; for others, the most complete account seems to have been written over thirty years ago by Lucille Whyburn [53]. The main elements of the method, however, can be gleaned from a quotation taken from an interview with one of Moore’s little known doctoral students, G. H. Hallett, fifty years after Hallett had been in his class [47, p. 84].

He taught in a very remarkable way. He didn’t give us any books. We **didn’t consult books** at all in that course. It was a course in **point set theory** and he gave us certain **axioms** to start with, and then we were asked . . . to **prove** certain theorems . . . We would work on the proofs and come back into class and he would ask how many people had the proofs and those who said yes were given a chance . . . to give their proofs. And the other members of the class listened carefully to see if they made any mistakes.

The items in boldface indicate that the students were handed sets of axioms and theorems. Their mission was to state precise definitions and to develop appropriate concepts in order to prove the theorems—without consulting anyone or anything. In short, the mission was to **do** mathematics.



Figure 1. R. L. Moore teaching in the classroom at the University of Texas. Taken from the MAA film *Challenge in the Classroom* [66].

A major misconception is to identify the Moore Method exclusively with Texas. For instance, a former *Focus* editor wrote, “R. L. Moore developed his approach to discovery learning from 1920 to 1969 at the University of Texas” [44]. In this paper we supply evidence that, contrary to this quotation, most of the Moore Method was developed before Moore joined the faculty at Texas in 1920. We hasten to emphasize, however, that *the popularity, success, and refinement of the method were achieved during Moore’s forty-nine-year tenure in Austin*. Nonetheless, almost all of the major elements emerged during the nine years Moore spent at the University of Pennsylvania (Penn), 1911–1920.

So deeply has the identification of the Moore Method with Texas become ingrained in our collective memory that few know that Moore taught at four institutions over a fifteen-year period from the time he received his doctorate until he returned to his home state. This is understandable. Contrast, for instance, a few of the names associated with the Moore Method at Penn (H. H. Mitchell, J. R. Kline, G. H. Hallett, Anna Mullikin, and Leo Zippin) with a few at Texas (R. L. Wilder, G. T. Whyburn, R. H. Bing, Mary Ellen Rudin, and Gail Young). Many readers will be able to identify the mathematicians in the Texas list, but only a few will have seen anything by or about the heretofore minor characters in the Penn list. One aim of this paper is to correct this vision.

We make no claim that Moore developed his eponymous method for training research mathematicians first at Penn. Although we supply evidence that it was operational by the time he left there, it is possible that he developed it beforehand. In fact, the paper begins with a brief overview of Moore’s activities up to 1911 with an aim toward suggesting early influences on his method. After that we describe in detail the crucial events that took place at Penn during the critical period 1911–1920. This includes Moore’s development of courses that later formed the core of his program and culminated in his first three doctoral students. The aim here is to demonstrate that the major elements of the Moore Method, as described in the defining work [53], were in place before Moore moved back to the Lone Star State. Next we examine how this early development in Philadelphia morphed smoothly to Austin in the next decade. In so doing we show how the early events paid dividends not only by establishing a Moore school of topology but launching international ties with the Polish school in Warsaw. (A recent paper by Albert C. Lewis [25] discusses the definition of “Moore school” and delineates some of its ties to the Polish school of topology.) Finally we describe ways in which Moore’s links to Penn played a role in the evolution of mathematics in America through the mid-1950s. Along the way we will meet some important, as well as some rank-and-file, characters in the history of mathematics in America during the period 1925–1975. We will also view the emergence of research institutions in the South, particularly Virginia and Tulane.

2. PRIMARY INFLUENCES. In this section we supply those aspects of Moore’s biography that played a role in the early development of his successful method of teaching. A fuller account can be found in the book by D. Reginald Traylor [47].¹ Robert Lee Moore (1882–1974) was born and raised in Dallas. Figure 2 shows the whippersnapper at an early age. He attended a private academy run by Waldemar Malcolmson, who provided an excellent foundation in English, history, and mathematics. Neither the teaching method nor Moore’s manner of learning are known from this

¹A negative review of this book by Paul Halmos [15] is based mainly on the reverential tone that characterizes many accounts of Moore and his method by members of the extended Moore family. However, this author has found [47] especially accurate regarding Moore’s life and work. A newer biography by John Parker, based on archival material not available thirty years ago, is in production.



Figure 2. R. L. Moore at a young age. Photograph courtesy of Louis A. Beerherl, Jr., and the Center for American History.

period, but some time by the age of fifteen he got the notion to attend the fledgling University of Texas. Although Texas had opened its doors only in 1883, it was modeled after the University of Virginia, so by 1898 its entrance requirements included knowledge of Greek or Latin. This policy caused him to withdraw from Malcolmson's academy to teach himself Latin.

During that year Moore also set about reading calculus because he enjoyed mathematics and wanted to extend his studies. So he borrowed a calculus book from Malcolmson but, dissatisfied with its imprecise language, wrote to the University of Texas requesting a copy of the book used there, Byerly's *Differential Calculus*.² In the film *Challenge in the Classroom* [66], which serves as his autobiography, Moore revealed

²William Elwood Byerly (1849–1935) earned the first Ph.D. in mathematics at Harvard in 1873. His *Elements of the Differential Calculus, with Examples and Applications* was published 1879–1901. For more information on calculus textbooks written in the nineteenth century, see the authoritative article by George Rosenstein [45].

his manner of learning mathematics at that time—one that he apparently came upon independently. He would read a theorem, cover its proof, attempt to prove it himself, and consult the proof only when absolutely necessary, and then one line at a time. Most mathematics instructors encourage their students to learn the subject this way; apparently Moore came upon it intuitively. This suggests one primary part of the Moore Method. Another is his determination not to obtain help from any other people or written materials. Learning mathematics for R. L. Moore meant being actively engaged and totally independent. The policy that the learner is primarily responsible for mastering new material would form a cornerstone of the Moore Method.

Even though Moore was not yet sixteen, he passed all four entrance examinations (in English, history, mathematics, and Latin) and matriculated at Texas in the fall of 1898. It didn't take him long to show his mettle. According to F. Burton Jones, within the first few weeks of the term, "when it became evident that calculus was not sufficiently challenging, Halsted transferred Moore to his course on projective geometry. Thus in his freshman year he was already in competition with juniors and seniors" [21, p. iv]. (We cite Jones because he "acted as an effective communications bridge between those trained in the Moore tradition and those with more conventional educations" [10, p. 43].) This incident shows that George Bruce Halsted (1853–1922) was responsible for three additional ingredients in the Moore Method: (1) recognizing talented, promising, hardworking students in a calculus class, (2) recruiting them to specialize in mathematics, and (3) directing their further studies. Halsted, "a man of character, strong opinions, and high ideals of scholarship" [21, p. iv], graduated from Princeton four years before earning the second Ph.D. from Johns Hopkins under J. J. Sylvester [42, p. 97]. After teaching at Princeton for five years he was appointed professor and department head at Texas in 1884. Moore thrived under Halsted's tutelage, earning B.S. and M.A. degrees in only three years. Although Halsted taught most of Moore's mathematics courses, Leonard E. Dickson (1874–1954) was his instructor in advanced calculus (elliptic integrals, gamma functions, Fourier series) and group theory [25].

Moore remained at Texas on a fellowship during 1901–1902, when a notable event sealed his academic fate. During that year Halsted asked Moore if he could prove that a particular axiom followed from other axioms in a work by the eminent German mathematician David Hilbert. Moore proved the redundancy at once, causing the duly impressed Halsted to dash off a note in this MONTHLY. Halsted wrote, "Early next morning he announced to me that he had demonstrated Hilbert's new axiom, thus reducing the Betweenness Assumptions from five to four . . . [T]he demonstration, as I have written it out from his oral communication, seems of most unexpected simplicity and elegance" [16, p. 98]. Although Halsted is listed as the sole author of the paper, he gives full credit to his protégé for the proof. (The books of Archibald [1, p. 243] and Traylor [47, p. 197] include the paper on Moore's list of publications; Lewis [25] notes that Moore did too.)

Though this was but one of several incidents that swirled around Halsted, we have not reached the end of the story. In the next issue of the MONTHLY, the chair of the mathematics department at the University of Chicago, E. H. Moore (1862–1932), published the text of a letter he had sent the nineteen-year-old R. L. Moore informing him that the result had already appeared in an article he had just published. E. H. acknowledged, however, that R. L. had proceeded independently. Moreover, he praised the teenager for "the delightfully simple proof of the redundancy" [31, p. 153].

Moore's fellowship did not continue the next year because he was no longer a student, but his success on the international stage emboldened Halsted to campaign for a teaching position for his protégé. When the president of the university chose someone

politically connected instead, Halsted wrote, “The bane of the state university is that its regents are the appointees of a politician” [17, p. 645]. He was fired within six weeks even though he had headed the department for eighteen years. Moore himself spent 1902–1903 as a high school teacher in Marshall, Texas. Not much is known about his high school teaching experience, including the manner in which he taught, but he never complained about it during the rest of his career.

In 1903 Moore was admitted to the graduate program at Chicago, the best in the country at the time and a cauldron of activity devoted to the axiomatic foundations of geometry. Given that E. H. Moore insisted upon the very highest standards of research, Chicago provided a perfect fit for R. L. Moore, who made an immediate impact. Oswald Veblen (1880–1960) had just completed his dissertation under E. H. Moore when R. L. Moore began his graduate studies in the fall term 1903. Yet by the time Veblen submitted his results for publication the next spring he acknowledged deep gratitude to R. L. Moore for critically reading parts of the manuscript [48, p. 344]. Conversely, Moore’s dissertation thanked Veblen for suggesting the undertaking, making numerous suggestions and criticisms, and providing much help “in the way of actual collaboration” [38, p. 488]. Figure 3 shows Moore during his time at Chicago.

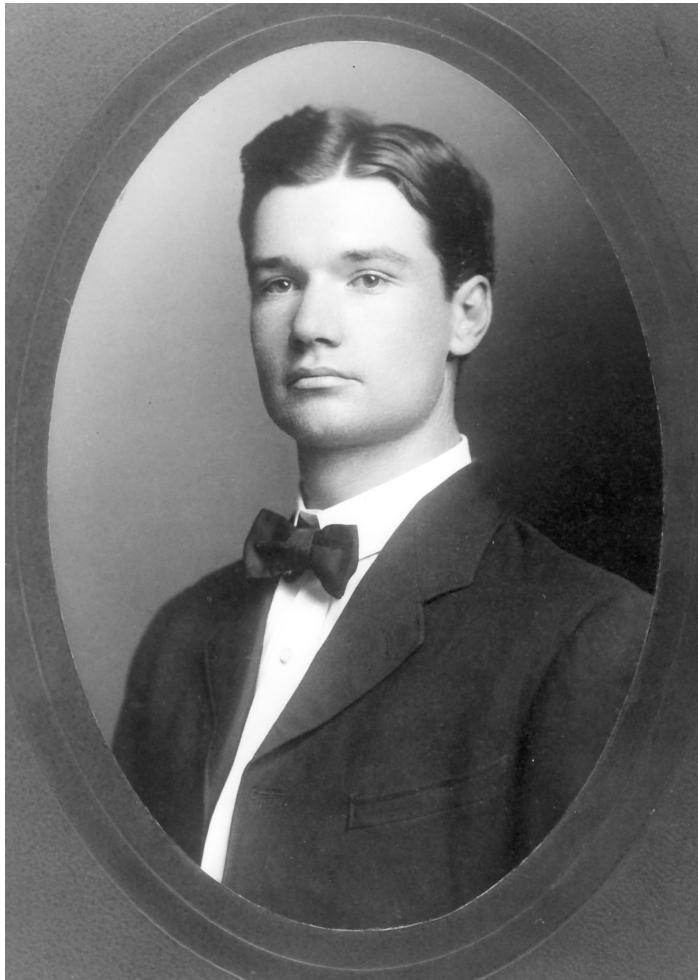


Figure 3. Photo of a dashing R. L. Moore c. 1904. Photograph courtesy of the Center for American History.

Upon receiving his Ph.D. in 1905, the twenty-two-year-old Moore embarked on an academic career, but he was unable to find a suitably accommodating environment for another six years. He held an instructorship at Tennessee for only one year, departing perhaps due to a disagreement with department head C. D. Schmitt³ over submitting grades (see [47, p. 65]) but more than likely because he realized he would be unable to teach graduate courses there. Besides, exciting stirrings were taking place at Princeton, where he joined Veblen in 1906. Both were hired by Henry Fine (1858–1928), who attributed his own interest in mathematics to G. B. Halsted (see [4, p. 196]). Moore had not yet published one paper. When he was at Tennessee, Veblen asked, “No doubt you are getting your geometry work into final form as quickly as possible? I am anxious to see that work come out as soon as possible” [A1].⁴ Moore was neither lazy nor undisciplined, so why didn’t he publish his dissertation right away? R. L. Wilder suggests a possible reason based on a letter from Veblen to Moore indicating that Moore had been attempting to find an axiomatization of the integers [58, pp. 75–76]. Perhaps this is what prompted Veblen’s letter to prod Moore into dropping the matter and finalizing his dissertation for publication. Aren’t all dissertations advisors anxious to see their student’s work appear in print? Unfortunately there are no extant copies of the paper Moore apparently submitted to the *Annals of Mathematics*, but he did publish two articles during his Princeton days, including his dissertation [38].

The preceptorship Moore held at Princeton during the period 1906–1908 might seem to have offered optimal conditions, but he was agitated by common exams. One notable aspect of the Moore Method is that a class moves with deliberate speed, with the students’ progress dictating the rate of new material, not some externally prepared syllabus. However, the overarching reason why Moore left Princeton was due not to differences in teaching philosophy, rather to the fact that he was regarded as eighth on the list of mathematicians there (see [26, p. 222]). So Moore sought a new position after the spring 1908 semester even though he had just taught a graduate course on the foundations of mathematics. He then taught at Northwestern from 1908 to 1911; little is known about this period, except that he had a very high teaching load and did not publish one paper.

Overall, the years 1905–1911 found Moore still in search of a supportive academic milieu. Fortunately he found it when he accepted an instructorship at Penn in 1911.

3. DEVELOPMENT. When R. L. Moore arrived at the University of Pennsylvania in 1911 he brought a bride of one year, a pedigree degree, and outstanding promise as a researcher, but a shallow record with no distinction at three universities and very little mathematics in published form. Yet he was only twenty-eight when he assumed a shared office in the basement of College Hall.⁵ By the time he left nine years later he had earned a reputation as one of the best mathematicians in the land. In this section we discuss the distinction Moore achieved while at Penn—publications, professional recognition, personal promotions, development of courses, influence on the teaching

³Cooper Davis Schmitt (1859–1910) earned a master’s degree at Virginia in 1884 and came to Tennessee five years later as professor of mathematics. He was the only professor in the department when he hired Moore in 1905; two years later he became a dean at the university. Except for a short note in this MONTHLY [46], and thanks he received for his support for the embryonic journal from its founder B. F. Finkel [9, p. 61], Schmitt’s name is associated only with problems in the *Annals* and this MONTHLY. He does not appear in Cajori’s book [7].

⁴This designation refers to the list of archival sources in section 6 of the present paper.

⁵Very few professors had private offices. Moore rented a house just a few blocks from campus, so he generally walked to College Hall to teach his classes. Office hours were held in the common room, but Moore did most of his work at home. Eckhart Hall at the University of Chicago in 1930 and Fine Hall at Princeton in 1931 established the policy of housing mathematicians in individual offices.

of colleagues, and, most importantly, his first successes as a dissertation advisor, including his mentoring.

The two papers Moore published before arriving at Penn were based on results he had obtained while still a graduate student. However, during nine years at Penn he produced an impressive output of seventeen papers. We do not analyze these papers; [58] provides a masterful account. Instead we highlight a few works that affected the development of the Moore Method.

Moore's 1915 paper "On a Set of Postulates Which Suffice to Define a Number-Plane" [36] illustrates the depth of the analytic ability he had begun to instill in his students. Ten years earlier Veblen had given the first complete, rigorous proof of the Jordan Curve Theorem [49]. However, Moore showed that its title "Theory of Plane Curves in Non-Metrical Analysis Situs" was incorrect by demonstrating that any space satisfying Veblen's axioms is homeomorphic to the Euclidean plane. Consequently all of Veblen's results take place in what we would call today metrizable topology and not "non-metrical analysis situs." (See [10, p. 45].)

Yet it was Moore's 1916 *Transactions* paper "On the Foundations of Plane Analysis Situs" [37] that firmly solidified his reputation as a first-rate researcher and later formed the basis for the Moore Method. We abbreviate it FPAS, though Moore used F.A. in his correspondence (see, for example, [51, p. 330]). Because of the success of the earlier paper [36], Moore turned his attention to topological characterizations of the Euclidean plane in FPAS and, indeed, in much of his subsequent work. Figure 4 reproduces a page demonstrating the precision of the language in the axiomatic approach that Moore espoused. Notice that *point* and *region* are undefined terms. Moore's regions would ultimately become open sets that form a basis for a topological space X . His goal was to find sets of axioms that would make X homeomorphic to the Euclidean plane \mathbb{R}^2 . Axiom 1 is complicated—condition (1) means that X has a countable basis (it also implies that X is separable), while condition (2) means that X is a Hausdorff space. Axioms 2 to 5 are simpler; they assert that X is locally connected (axiom 2), co-connected (axiom 3), and locally compact (axiom 4) but not compact (axiom 5). Moore proceeded to prove fifty-two theorems from this set of five assumptions. (See [22] for additional remarks on FPAS.)

Moore's career quickly benefited from these stunning successes. The publication of FPAS and a brief summary that preceded it in the *Proceedings of the National Academy of Sciences* led to his promotion to assistant professor in 1916. That year saw him notch two other successes, both of which concern his doctoral students. To set the stage for the way Moore trained them we mention two courses that he offered. During the academic year 1912–1913 he taught a graduate course entitled "Foundations of Mathematics." The following year, slightly before the publication of Hausdorff's landmark book on topology, *Grundzüge der Mengenlehre*, Moore instituted a new course "Theory of Point Sets," whose catalog description reads:

Theory of sets of points in metrical and in non-metrical spaces. Contributions of Fréchet and others to the foundations of point set theory. Content and measure. Jordan curve theory and other applications.

These two courses formed the cornerstone of a longer list of courses that came to comprise the Moore Method when it was fully developed.

John Robert Kline (1891–1955) enrolled in the graduate program at Penn in the fall of 1913, one year after graduating from Muhlenberg College. Neither the Registrar's Office nor the archives at the University of Pennsylvania contains records from Moore's classes, but there is convincing evidence in the Moore Archives at the Center

can be potentially descriptive without being separable and, indeed, *metrical* in the sense that it is in a one to one continuous correspondence with an ordinary euclidean space of two dimensions.

2. AXIOMS AND DEFINITIONS

I consider a class, S , of elements called *points* and a class of sub-classes of S called *regions*, subject to a set of postulates (axioms) as described below. Before stating these axioms I will define certain terms that will be used.

DEFINITIONS. A point P is said to be a *limit point* of a point-set M if, and only if, every region that contains P contains at least one point of M distinct from P . The *boundary* of a point-set M is the set of all points $[X]$ such that every region that contains X contains at least one point of M and at least one point that does not belong to M . If M is a point-set, M' denotes the set of points composed of M plus its boundary. If R is a region, the point-set $S - R'$ is called the *exterior* of R . A point in the exterior of R is said to be *without* R .

A set of points is said to be *connected* if, however it be divided into two mutually exclusive subsets, one of them contains a limit point of the other one.

A set of regions K is said to *cover* a point-set M if every point of M belongs to at least one region of the set K . If for every infinite set of regions K covering the point-set M there exists a finite subset of K that also covers M then M is said to *possess the Heine-Borel property*.

AXIOM 1.* *There exists an infinite sequence of regions, K_1, K_2, K_3, \dots such that (1) if m is an integer and P is a point, there exists an integer n , greater than m , such that K_n contains P , (2) if P and \bar{P} are distinct points of a region R then there exists an integer δ such that if $n > \delta$ and K_n contains P then K'_n is a subset of $R - \bar{P}$.*

AXIOM 2. *Every region is a connected set of points.*

AXIOM 3. *If R is a region, $S - R'$ is a connected set of points.*

AXIOM 4. *If R is a region, R' possesses the Heine-Borel property.*

AXIOM 5. *There exists an infinite set of points that has no limit point.*

AXIOM 6. *If R and \bar{R} are regions and P is a point in \bar{R} and on the boundary of R , then there exist in \bar{R} two regions K and \bar{K} such that \bar{K} contains P , K lies in R and all those points of the boundary of R that lie in \bar{K} are points also of the boundary of K .*

AXIOM 7. *If R and \bar{R} are regions and P is a point in \bar{R} and on the boundary of R , then there exist in \bar{R} two regions L and \bar{L} such that \bar{L} contains P , L lies*

* There is a certain amount of resemblance between Axiom 1 and Veblen's Postulate of Uniformity. Cf. O. Veblen, *Definition in terms of order alone in the linear continuum and in well-ordered sets*, these *Transactions*, vol. 6 (1905), p. 169.

Figure 4. The axiomatic approach of R. L. Moore.

for American History in Austin suggesting his attendance in the point sets course. For instance, in a letter written about ten years later Kline recalled, “In 1914 when I was in your class . . .” [A2]. On June 1, 1915, an Allentown newspaper announced Kline’s wedding and added, “Following the ceremony, Prof. Robert Lee Moore . . . will tender a luncheon to the newly married couple” [A3]. Such an action on Moore’s part may seem to contradict the folklore that Moore, like many dissertation advisors, was opposed to marriage before the completion of all requirements for the degree. However, Kline had presented his results at a meeting of the American Mathematical Society (AMS) in New York five weeks earlier. What excitement must have preceded his talk! The audience included a veritable Who’s Who from the Chicago school—G. D. Birkhoff, E. H. Moore, R. L. Moore, H. E. Slaught, and O. Veblen. (See [8] for complete details of the meeting.)



Figure 5. J. R. Kline. Photograph courtesy of the American Mathematical Society.

Kline became Moore’s first success story in 1916 in two different ways. The more important was that when he graduated in 1916 he became Moore’s first Ph.D. student; his dissertation also appeared that year [23]. Figure 6 reproduces its first page to demonstrate how Moore’s students inherited his manner of writing. (Compare Figures 4 and 6.) Its first sentence attests to the influence of G. B. Halsted on yet another generation; a footnote on the second page attests to the legacy of Veblen’s work from the University of Chicago.

Another success involving Kline awaited Moore in 1916. Oswald Veblen had informed Moore that his paper on an axiomatization of the positive integers had been rejected by *Annals* editor Edward Vermilye Huntington (1874–1952) because he probably did not fully comprehend the wording of Moore’s deep (and verbose) axioms. Karen Parshall recently observed, “Especially during the first five years of the new century, competition was stiff between Huntington and the Chicagoans” [41, p. 221]. A joint paper by Huntington and Kline from the triumphant year 1916 suggests that the rivalry within the American school of postulate theorists—pitting easterners (notably Harvard’s Huntington) against the Chicago contingent (including E. H. Moore,

DOUBLE ELLIPTIC GEOMETRY IN TERMS OF POINT AND ORDER ALONE.*

BY J. R. KLINE.

1. Introduction.

In his *Rational Geometry*,† Halsted built up two-dimensional double elliptic geometry, in terms of the undefined symbols point, order, association and congruence. In the present paper I propose a categorical set of ten mutually independent axioms for three-dimensional double elliptic geometry. Only the symbols point and order are undefined.

I wish to express my deep gratitude to Professor Robert L. Moore, who suggested the problem of this paper and aided me continually in its preparation.

2. Axioms and Definitions.

AXIOM I. If A is a point, then there exists at least one point A' , different from A , such that $AA'C\ddagger$ is false for every C .

DEFINITION 1. If A and B are distinct points such that ABC is false for every C , then B is said to be an *opposite* of A . If A is a point, A' denotes an opposite of A .

AXIOM II. There exist two distinct points A and B such that B is not an opposite of A .

AXIOM III. If ABC , then CBA .

AXIOM IV. If ABC and A' is an opposite of A , then $A'CB$.

AXIOM V. If ABC and ACD , then ABD .

DEFINITION 2. The *line* $AB(A \neq B \neq A')$ is the set of all points $[X]$ such that $X = A$, $X = A'$, $X = B$, $X = B'$, XAB , AXB , ABX or $AX'B$. The set of all points $[X]$ such that AXB constitute the *segment* AB . A and B are the *end points* of this segment.

AXIOM VI. If there exist three distinct points, no two of which are opposites, then there exist three distinct points A , B and C such that A is not an opposite of B and such that C is not on the line AB .

AXIOM VII. If D is not on the line AB and C and E are such that ABC and BED and if there exists F such that DFA , then there exists G such that CEG and DGA .

* Presented to the Society, April 24, 1915.

† G. B. Halsted, *Rational Geometry*, John Wiley and Sons, pp. 212-247.

‡ The abbreviation " $AA'C$ " signifies that the points A , A' and C are in the order $AA'C$.

O. Veblen, L. E. Dickson, and R. L. Moore)—extended through the second decade of the twentieth century. Given Moore’s role vis-à-vis Huntington, such collaboration seems startling initially. Upon closer inspection, however, the second author is listed as J. Robert Kline, the only time this form appears in print in any document bearing Kline’s name. Not once in the massive Kline files archived in Austin and Philadelphia did he sign his name this way. Fortunately an otherwise routine footnote suggests how the paper, read at an AMS meeting on September 5, 1916, might have evolved into a joint effort [20, p. 301]:

The parts of the paper which do not involve the postulates here numbered 5 and 8 were presented by Professor Huntington at the meetings of December 31, 1912, and April 26, 1913. The necessity of adding postulates 5 and 8 was kindly pointed out by Professor R. L. Moore, and all the theorems and examples which involve these two postulates are due to Dr. Kline.

This comment and the form of Kline’s name suggest that Huntington had written the paper alone but felt obligated to share authorship with a student of a postulational rival due to the protégé’s painstaking analysis. R. L. Moore had attended Huntington’s 1913 presentation and must have subsequently informed J. R. Kline, who then subjected Huntington’s postulates to the scrutiny of someone schooled in the Moore Method. One can imagine Kline and the competitive Moore celebrating their victory over their prestigious foe in the basement of College Hall. Cast in this light, perhaps the word *kindly* in Huntington’s footnote would be better served by *gleefully*.

J. R. Kline stayed at Penn for two more years—the first on a fellowship and the second as an instructor—before moving to Yale and Illinois for one year each. He returned to Penn in 1920 and remained there for the rest of his career. He penned only one other joint paper—with R. L. Moore—and it turned out to be Moore’s solitary coauthored work [39]. Collaboration was not in the Moore family lexicon.

George H. Hallett, Jr. (1895–1985) became R. L. Moore’s second Ph.D. student when he received his degree in 1918. In his dissertation, which like Kline’s dealt with geometry, he wrote, “This problem was suggested by Professor Robert L. Moore, to whom I am likewise indebted for many valuable suggestions in its solution” [13, p. 186]. Knowing Moore’s uncanny ability to pose questions at the right time, it seems safe to say that “suggestions” refers to questions to be resolved by proof or counterexample. Although much of the early part of the thesis relied on Veblen’s work from 1904 and Kline’s recently completed dissertation, Hallett also cited Halsted’s pioneering role in introducing a certain double elliptic geometry, thus attesting to the continuing influence of G. B. Halsted on the development of mathematics in the country.

Hallett inherited his mathematical genes: his father was a colleague of Moore’s who had obtained his own Penn doctorate in 1896. The younger Hallett spent a lifetime in government work; his 1937 book [14] on proportional representation became a landmark in political theory. Upon his death the American Political Science Association established the George H. Hallett Award, presented annually to the author of a book (published at least ten years earlier) that had made important contributions to the literature on representation and electoral systems. It was Hallett who made the statement that began this paper in a 1971 interview, fifty-five years after taking Moore’s classes. He credited the Moore Method for much of his success [47, p. 86]:

Dr. Moore’s method of teaching brought out what appeared to me to be the two most important faculties in mathematical research... imagination and critical analysis... I think such success as I’ve had in the work I’ve done in the field of government probably has a good deal to do with that—because they don’t catch me up very often in theories of logic in bills, or different parts of bills, that don’t hang together.

Hallett's attribution confirms that, even before reaching Texas, R. L. Moore exerted a profound influence on students in fields outside mathematics. Apparently Moore also affected the teaching methods of some of his colleagues. G. H. Hallett described a course taught by Howard Hawks Mitchell (1885–1943), who, like Moore, had come to Penn as an instructor in 1911 [47, p. 85]:

Professor Mitchell . . . gave us this book, but asked us to go through all the proofs that were given and find out whether they were watertight proofs or not and if not, why not Actually this course had many elements of the other course. We'd discuss the matters in class together.

Hallett's account recalls a course taught by H. H. Mitchell using a real variables textbook written by Yale's James Pierpont [43]. The "other course" in the penultimate sentence is most likely Moore's "Foundations of Mathematics" since he offered it when Hallett matriculated in 1916. Consequently at least one mathematics professor adapted Moore's manner of teaching to accommodate his own personality and circumstances, resulting in the earliest known example of what today is termed a modified Moore Method course. Incidentally, Mitchell was an associate editor of the *Annals* when Kline and Hallett published their dissertations there.⁶

R. L. Moore produced one other Ph.D. student at Penn, Anna Margaret Mullikin (1893–1975), who enrolled in the graduate program when Kline and Hallett departed in 1918. Moore recognized her ability and—more importantly for his method—her potential at once. His first female doctoral student (of five altogether) made such vast strides that she read a paper at an AMS meeting in October 1919, a month into her second year of graduate studies. When later asked to recall Moore's method of teaching, she emphasized his insistence that students settle questions without assistance from any sources—human or written [47, p. 87].⁷

He had his work all published . . . and people would go and look it up in the library and he didn't want them to do that. He wanted them to work it out themselves. And he put them out of class if he found out that they were cheating. (In one class) there were three of us, but he put everybody out but me. One was a Catholic nun and she tried to get help from me and he put her out. He said if she needed help she didn't belong in his class.

Although Mullikin had made substantial progress on her dissertation before Moore returned to Texas at the end of her second year, she too moved to Austin. She completed it that year and returned to Penn in the fall of 1921 to satisfy residency requirements. J. R. Kline arranged a half scholarship to help defray the cost of tuition and then took care of the administrative details necessary for the awarding of her Ph.D. degree in June 1922. As we will see, she continued to play a role in the fully mature Moore Method of the 1950s.

On one hand Anna Mullikin serves as a transitional figure in the Moore Method from its developmental stage to one of complete maturity. On the other, Mullikin represents a radical break from Kline and Hallett in three notable ways. For one, her dissertation [40] dealt with topology, not geometry. Second, it was inspired not by G. B. Halsted, rather by the Polish topologist W. Sierpinski. Third, it was published in the

⁶The *Annals* has been published at Princeton since 1911. Mitchell was one of four cooperating editors when Kline's dissertation appeared. R. D. Carmichael of Illinois was the only other one not at Princeton at that time, but he had earned his Princeton Ph.D. in 1911 as G. D. Birkhoff's first doctoral student.

⁷Moore himself would later pay a price for not reading more widely. In a note referring to an earlier paper from *Fundamenta Mathematicae* he admitted, "Professor M. Fréchet has kindly called my attention to the fact that Theorems 1–3 of my paper are not new I regret having overlooked these results of Gross and Fréchet and I wish to thank Professor Fréchet for calling my attention to them" [32, p. 374].

Transactions as opposed to the *Annals*. Even here the symbiotic relationship between Moore and his former Penn colleague H. H. Mitchell was evident, because both were associate editors of the *Transactions* when Mullikin's dissertation appeared. While Mullikin held an instructorship at Texas during 1920–1921, Moore was imposing his teaching methodology on a new batch of students in a completely different location. As at Penn, he found a setting that allowed him to refine his method unimpeded.

4. IMPACT. Reports in this MONTHLY reveal a hitherto overlooked platform that provided a springboard for some of Moore's recruitment of students into his budding program. An undergraduate mathematics club called The Pentagonagram had been formed in 1916 by A. A. Bennett "to assist in promoting the interests of mathematics among the students of the University of Texas" [2, p. 273]. The Pentagonagram was part of a wave of undergraduate mathematics clubs that arose in the 1910s; there were twenty-six nationwide by the time Bennett founded it.⁸ The club met biweekly for lectures and problem-solving sessions, with speakers alternating between faculty members and students. The name Pentagonagram was chosen for two reasons, one being its obvious connection to mathematics. The club's initial student speaker stated that the second reason "was because of its significance to every Texan, Texas being called the Lone Star State, there being five letters in the word, Texas having fought under five flags and having served under five governments, and the State seal being a five-pointed star artistically wreathed and engraved" [2, p. 273]. In 1917 Bennett anticipated Moore's arrival on campus by three years when he lectured to the club on analysis situs, a topic that reflected his own tie to Oswald Veblen.⁹

R. L. Moore participated in The Pentagonagram from the moment he appeared on campus, speaking on Halsted's specialty—non-Euclidean geometry—at the first meeting of 1920–1921. The next year he spoke on another subject dear to his heart, "Fallacies of Elementary Geometry." One of the students smitten by him was Renke Gustav Lubben (1898–1980), then a senior and one of two members of the club's executive council.¹⁰ Six months before Moore came to Texas, Lubben presented a student paper "Codes and Ciphers," which was subsequently published in the *Texas Mathematics Teachers' Bulletin* [27]. He served as a tutor (a teaching assistant) during 1921–1922 but became an instructor for the next four years, all the while pursuing a Ph.D. degree under Moore, graduating in 1925 with a dissertation on geometry inspired by the work of Halsted (like Kline and Hallett). In the thesis, Lubben credited Moore for developing his research ability: "I wish to thank Professor R. L. Moore for interesting me in the subject of the Foundations of Mathematics, and for arousing in me a desire to

⁸Norbert Wiener lectured on the fourth dimension to such a club in the first semester of his one-year stint at the University of Maine 1916–1917 [5, pp. 14–15]. The Vinculum, founded at Penn for women in 1917, evolved into an inclusive Pi Mu Epsilon chapter four years later, with H. H. Mitchell playing a prominent role. Further details about undergraduate mathematics clubs up to the end of 1918 are given in [3].

⁹See [64] for a fuller account of Albert Arnold Bennett (1888–1971), who spent most of his career at Brown University. He accepted an adjunct professorship (an assistant professorship today) at Texas one year after receiving his Princeton Ph.D. under Oswald Veblen in 1915. In recommending him for the position Veblen wrote that Bennett "has wider knowledge of mathematics than any man of his age whom I know" (as quoted in [26, p. 224]). There is no evidence that Bennett met Moore when Bennett was at Princeton and Moore in Philadelphia. In World War I, Bennett served under Veblen at Aberdeen Proving Ground with Gilbert Bliss and Norbert Wiener [12]. The book [63] describes Bennett's founding with H. H. Mitchell of the Philadelphia Section of the MAA in 1926.

¹⁰The other member was Goldie Prentis Horton (1887–1972), the first person to earn a Ph.D. degree in mathematics at the University of Texas. She wrote her dissertation in 1916 under Milton Brockett Porter (1869–1960) and joined the Texas faculty as an instructor in 1917. In 1934 Porter and Horton published a book on analytic geometry; they celebrated its publication by getting married. Porter retired in 1945 and Horton-Porter in 1966.

contribute to the extension of this field” [28, p. 37]. Lubben was awarded a National Research Council (NRC) fellowship for postdoctoral study in Europe for the academic year 1926–1927, choosing Göttingen for two reasons: it housed arguably the most accomplished mathematics faculty in the world, and J. R. Kline was already there among the first group of Guggenheim Fellows.¹¹

Upon returning from abroad Lubben accepted a position at the University of Texas, where he remained for the rest of his career, becoming the only Moore student to spend his entire career there. During his thirty years on the faculty Lubben published six research articles but produced no doctoral students, perhaps, as Albert Lewis observed in [25], due to illness. According to R. L. Wilder, another reason might be that “Moore made it a policy not to let anyone teach courses in his field of point set topology. As a matter of fact, if a student who had earned his degree under Moore didn’t go on to another institution he just stayed at Texas and had to teach other kinds of courses” [56, p. 197]. Apparently such ownership of courses in one’s field was characteristic of many professors at that time. While R. G. Lubben is a mostly forgotten mathematician, the same cannot be said of the two Moore students surrounding him, R. L. Wilder and G. T. Whyburn. Wilder became Moore’s first doctoral student at Texas when he earned his Ph.D. in 1923, only one year after Mullikin. In an obituary of Moore, Wilder wrote, “An essential part of the [Moore] method was Moore’s ability to search out and recognize creative ability” [57, p. 417], a description that refers to Wilder, Lubben, and Whyburn as much as Moore’s three Penn students.

Raymond Louis Wilder (1896–1982) enrolled in the graduate program at the University of Texas in 1921—the same year as Lubben—after receiving a master’s degree from Brown. He intended to complete a program in actuarial mathematics under Edward Lewis Dodd, who had first taught the subject there in 1912 and proceeded to earn nationwide recognition, but Dodd suggested he take Moore’s course in order to gain



Figure 7. R. L. Wilder. Photograph courtesy of the American Mathematical Society.

¹¹In informing Moore of his selection, Kline boasted proudly of his appointment [A4]. In subsequent correspondence Kline reported to Moore on Lubben’s activities, thus illustrating the supportive bond that was emerging among Moore school participants. Moore himself never left the United States.

a better acquaintance with pure mathematics. The first meeting between Wilder and Moore was inauspicious. In remarks at a presentation breakfast honoring the memory of Moore in 1976, Wilder recalled, “I soon realized that he was very negative about my enrolling in his course I had two counts against me, as I analyzed it later. One was that I was a Yankee. The second was that I was an actuarial student” [52, p. 5]. Initially Moore ignored Wilder in class until the student proved the *pons asinorum*, Theorem 15 in FPAS: if A and B are distinct points of a domain M , there exists a simple continuous arc from A to B that lies wholly in M [37, p. 136]. Yet in spite of the rocky start Wilder had smooth sailing after solving a problem that had eluded J. R. Kline. Moore encouraged him to write up his solution promptly, whereupon “Moore cut through red tape and arranged for Wilder to take his [language and qualifying] exams after the deadline” [6, p. 117]. When Wilder received his Ph.D. in 1923 he was only the second person to get the degree in mathematics at Texas (see footnote 10). Wilder’s dissertation appeared in *Fundamenta Mathematicae* two years later [54]. By then he had already published two papers in the celebrated Polish journal. On a related note, Lubben’s dissertation also appeared in *Fundamenta*, but not until three years after it was accepted for his Ph.D.

Wilder enjoyed a particularly distinguished career in academia, associated with the University of Michigan from 1926 until his retirement at age seventy in 1967. He presented invited AMS lectures on three separate occasions: a special symposium in 1932, as Colloquium Lecturer in 1943, and as Gibbs Lecturer in 1969. But perhaps his most distinguished lecture was the one he was invited to present at the AMS Semi-centennial Celebration in 1938. (J. R. Kline was on the three-person Subcommittee on the Program for the meeting.) Wilder’s address, “The Sphere in Topology,” appeared in the two-volume set devoted to the Society’s first fifty years. In the introduction he wrote [60, p. 136]:

Probably no branch of mathematics has experienced a more surprising growth than . . . Topology or Analysis Situs It has provided a tool for classification and unification, as well as for extension and generalization It is with pardonable pride that one can point to the part which American mathematicians have played in this development. Although his name will not appear again in this monograph, we may well ponder how much this was due to that great American mathematician, E. H. Moore, by whose students, particularly R. L. Moore and O. Veblen, the actual beginnings of Topology in this country were made.

It is particularly impressive that articles by Wilder were chosen for both the two-volume semicentennial celebration of 1938 and the three-volume centennial celebration of 1989. He used a modified Moore method to produce twenty-five Ph.D.’s, even influencing other graduate students (notably Norman Steenrod) and undergraduate students (Edward Griffith Begle, who became interested in topology in Wilder’s undergraduate course). Wilder’s course on foundations became one of the most popular at Michigan.

The second half of the 1920s featured a steady flow of Moore and Kline students between Austin and Philadelphia. Perhaps the most unusual situation surrounded W. L. Ayres (1905–1976), who started out under Moore at Texas but transferred to Penn to continue under J. R. Kline because Moore felt that he would derive greater benefit there. Ayres earned his Penn Ph.D. in 1927, but when he won a National Research Fellowship the next year he took it in Austin. He is probably the only mathematician to be elected president of the American Rose Society and a Rotary Club (Lafayette). Another Moore student, John H. Roberts (1906–1997), who completed his dissertation in 1929, spent a postdoctoral year working with Kline at Penn. The Moore-Kline continuum also proceeded in the reverse direction, with three of Kline’s doctoral students

(in addition to Ayres) spending postdoctoral years with Moore: Harry Gehman (Ph.D. 1925), Norman Rutt (Ph.D. 1929), and Leo Zippin (Ph.D. 1929). Solomon Lefschetz tried to attract Gehman to Princeton for the year, but he chose Texas instead [A5]. However, R. L. Moore did not devote much time to postdoctoral students, focusing his energies instead on developing the skills of the talented neophytes whose programs of study he was directing.

A few years after Moore arrived at Texas he invited E. W. Chittenden (1885–1965) to teach a summer course in topology. The conduct in the class illustrates how adept Moore’s talented students had become at proving theorems. “[A]fter a few days Chittenden complained to Moore that they (the students) didn’t know anything. Moore suggested that they be given some theorems to prove. After a couple of weeks, Chittenden complained that they had proved all the theorems he had planned for the summer” [22, p. 101]. The class included G. T. Whyburn, his brother W. M. Whyburn, and G. T.’s future wife, Lucille Smith. All three hailed from Lewisville, Texas (located twenty-five miles north of Dallas), so when Gordon Thomas Whyburn (1904–1969) earned his Ph.D. he became Moore’s first Texas-born graduate. When he enrolled in Moore’s calculus class his major interest was chemistry, a field in which he earned an A.B. in 1925 and an M.A. in 1926, but he continued to take courses with Moore, who constantly pressured him to switch to mathematics because of the high level research he was carrying out. Therefore when Whyburn did change fields it took only six months before he presented his first paper to an AMS audience in Chicago and another six before he earned his Ph.D. in 1927. His dissertation was published in the *Transactions* that year, which seems impressive enough, but he had already published three papers by then, one in the prestigious *Proceedings of the National Academy of Sciences*. This record provides additional evidence of the effectiveness of the Moore Method in its maturing stage. Whyburn acknowledged his debt to Moore: “It is his stimulating personality, constant encouragement, and many helpful suggestions and criticisms which have attracted my interest to this field of mathematics and have made possible the solution of the problems treated in this paper” [50, p. 369].



Figure 8. G. T. Whyburn. Photograph courtesy of the American Mathematical Society.

Whyburn stayed at Texas the next two years as an adjunct professor and then spent a year in Europe on a Guggenheim fellowship, working with Hans Hahn and Karl Menger in Vienna and with Waclaw Sierpiński, Casimir Kuratowski, and Bronislaw Knaster in Warsaw. (For more on these European sojourns see [51].) Upon returning to the States Whyburn spent four years at Johns Hopkins before accepting a special offer at the University of Virginia, whose three professors of mathematics were about to retire. An obituary of Whyburn—written jointly by one of his students, E. E. Floyd, and one of Moore’s, F. B. Jones—aptly described the situation he faced [11, p. 58]:

The university sought to use the opportunity to establish a first-rate research and graduate department. There were very few Ph.D. programs in the South, most of them were of token character, and only the programs at Texas and Rice were in any sense distinguished. Whyburn became excited about the prospects The situation at Virginia inspired from Whyburn an ideal plan to fit the circumstances. One would get together a few young and congenial mathematicians of topflight accomplishments The plan worked beautifully.

As professor and chair of the department, Whyburn set about reinvigorating mathematics at Virginia with Texas as a model, which is somewhat ironic because the University of Texas had been founded with the University of Virginia as a model. The next year he hired Edward James McShane (1904–1989), and the two developed an excellent graduate program while remaining at the University for the rest of their careers. The quality of the program can be seen in the careers of two Whyburn students from that early period, A. D. Wallace (Ph.D. 1939) and John L. Kelley (Ph.D. 1940). We will meet Wallace shortly. For now we note that Kelley’s *General Topology* became a bible for aspiring topologists and analysts from the time it first appeared in 1955.

Whyburn played an instrumental part in Mullikin’s theorem, a result that signaled the beginning of international competition and cooperation between the Austin and Warsaw schools of topology. The Polish mathematician Stefan Mazurkiewicz was the first person to single it out in his 1924 paper “Remarque sur un théorème de M. Mullikin” [30]. In the same volume of *Fundamenta Mathematicae*, R. L. Moore made use of the result in solving a problem posed by Waclaw Sierpiński. Moore indicated that another paper in that volume had informed him that Zygmunt Janiszewski had actually proved the result several years before Mullikin, but it was written in Polish in a journal dedicated to mathematical physics. Moore himself had not seen the original paper, and would not have been able to read it even if he had, so he acknowledged that neither Mullikin nor he “was aware that the proposition had already been proved” [34, p. 190]. The result first appeared in an American publication at about the same time when Moore cited “a theorem of Miss Mullikin’s” [33, p. 169]. In 1927 Whyburn’s dissertation made critical use of the theorem on four separate occasions [50, pp. 397–400]. The recognition of the efficacy of the theorem aroused considerable interest among researchers inside and out of the Moore school. Internally, in 1931 R. L. Wilder preceded a reformulation of the theorem with a phrase favored by many mathematicians, “it is well known that” [55, p. 40]. In the interim, Leo Zippin (1905–1995) was analyzing and extending Mullikin’s results as part of his research program at Penn under J. R. Kline. The title of Zippin’s 1929 dissertation, “A Study of Continuous Curves and Their Relation to the Janiszewski-Mullikin Theorem,” reflects the origin of the result [61]. He wrote another article the next year shortening the name to JMT, an abbreviation that suggests yet another level of acceptance [62, p. 339].

Mullikin herself played a decisive role in one other connection between R. L. Moore and J. R. Kline. In spite of the international acclaim her work received, she remained a high-school teacher in Philadelphia for the rest of her career. When her gifted pupil Mary-Elizabeth Hamstrom received her diploma in 1944, Mullikin directed her to J. R.

Kline at Penn. Yet four years later Hamstrom had given little thought to life after the B.A., so when Kline recommended she pursue graduate studies at Texas she admitted that it was “a thought that hadn’t occurred to me” [18, p. 295]. Hamstrom did apply—and was admitted at once. Wanting to be fully prepared for the beginning of classes in September 1948 she wrote to R. L. Moore in late April asking what material she should read over the summer. His six-page reply has become the classic document describing the philosophy of the Moore Method, as evidenced by its inclusion among the documents published to celebrate the one-hundredth annual meeting of the AMS [18].



Figure 9. M.-E. Hamstrom. Photograph courtesy of Gayle Ball and the Center for American History.

The Mullikin-Kline-Hamstrom chain was not R. L. Moore’s final link with the University of Pennsylvania, where J. R. Kline had become chair in 1940. One year later Kline hired the aforementioned Alexander Doniphan Wallace (1905–1985), who had earned his Ph.D. in 1939 at the University of Virginia under G. T. Whyburn. Wallace remained at Penn until 1947 and “it was during these years that [he] developed his expertise in algebraic topology” [19, p. 2]. Later Wallace kept to his often-stated goal of building up mathematics in the South by helping W. L. Duren revivify the graduate program at Tulane University. Wallace no doubt was influenced by G. T. Whyburn, who had accomplished that same feat at Virginia. One year after Wallace departed Philadelphia for New Orleans, J. R. Kline hired Moore’s newly minted Ph.D., Richard Davis Anderson. This appointment turned out to be serendipitous for Lida Kittrell Barrett, who had studied under R. L. Moore until 1951, when her husband of one year accepted a position at Delaware. So, as Barrett wrote, “Moore and Kline worked out

an arrangement for me to continue my work at Penn” [A6]. However, one day after Barrett submitted the first complete draft of her thesis, Kline suffered a heart attack that prevented him from seeing the project to completion. Fortunately, R. D. Anderson was able to help her polish and refine her dissertation, which she completed in 1954. Barrett later became president of the MAA, a position also held by five other Moore students: R. L. Wilder, G. S. Young, R. H. Bing, E. E. Moise, and R. D. Anderson.

5. CONCLUSION. R. L. Moore’s relationship with the University of Pennsylvania began during his 1911–1920 tenure, when he developed his famous teaching method that resulted in three doctoral students, namely, J. R. Kline, George Hallett, and Anna Mullikin. Several events from Moore’s first twelve years at Texas attest to the foundation of a Moore school: the exchange of postdoctoral students under Moore and Kline, the close connection of dissertations written under Moore at Penn and at Texas, and the involvement of both Kline’s students and Moore’s students in applying and generalizing Mullikin’s theorem. The Mullikin-Kline-Hamstrom and Kline-Anderson-Barrett connections brought Moore’s relationship with Penn full circle. Those events reflect the extent to which the University of Pennsylvania played a central role in the Moore Method and the Moore school beyond when Moore left Philadelphia, but they came to an end with Kline’s untimely death in 1955.

Moore’s enduring fame with his teaching method, as opposed to his accomplishments in point set topology, rests on the quality of his fifty doctoral students. According to one biographer, this group “is regarded by many mathematicians outside the Moore school as the most distinguished group of mathematicians in the United States to have been taught by the same person” [24, p. 652]. Although forty-seven of those students were produced at Texas and are mainly responsible for the renown of the Moore Method, our account indicates that the main features of the method were in place by the time he arrived there in 1920. While the Moore Method produced internationally recognized scholars R. L. Wilder and G. T. Whyburn during the 1920s, it evolved subtly after the appearance of his trailblazing book *Foundations of Point Set Theory* in 1932 [35]. Altogether Moore directed forty-one doctoral students after that time, with their most important definitions, concepts, methods, and theorems finding a place in the revised edition published thirty years later. One vital aspect of the Moore Method did not change, however—independence. Moore is said to have permanently borrowed the library’s only copy to prevent his students from consulting it.

When was the name “Moore Method” coined? The earliest occurrence in print that we were able to locate occurred in a 1967 article in this MONTHLY by R. L. Wilder. His statement that “most of us have heard of the ‘Moore Method’ of teaching” [59, p. 125] suggests that the term had been widely used before then. Later that year the MAA film *Challenge in the Classroom: The Method of R. L. Moore* made its debut [66]. In 1968 the famed mathematician and historian of mathematics Kenneth O. May (1915–1977) cited the “Moore method” as an example of an effective pedagogical technique in an article on undergraduate research based on his ten-year experience at Carleton College [29, p. 71]. However, the defining moment for the Moore Method occurred in 1970 with the publication of “Student Oriented Teaching—The Moore Method” by Lucille S. Whyburn [53].

As this paper is being written, the Educational Advancement Foundation is completing its move into the Moore House, which it has fully restored. The Moore House is located about a mile from the Moore Building, a modern, high-rise structure housing the departments of mathematics, physics, and astronomy. The second level of the Moore House will contain seminar rooms for discussions on active learning. Although

the house has no basement, this paper has argued that its keystone foundation lies 1700 miles northeast of Austin.

6. ARCHIVAL SOURCES. In addition to published items, the author has drawn material from a number of important archival sources:

- A1. O. Veblen to R. L. Moore, April 9, 1906, Box 23, R. L. Moore Archives, Center for American History, The University of Texas at Austin.
- A2. J. R. Kline to R. L. Moore, undated and incomplete letter, Box 20, R. L. Moore Archives, Center for American History, The University of Texas at Austin.
- A3. Anon, College day romance to culminate today, June 1, 1915, J. R. Kline file, Archives of the University of Pennsylvania.
- A4. J. R. Kline to R. L. Moore, July 6, 1925, Box 20, R. L. Moore Archives, Center for American History, The University of Texas at Austin.
- A5. S. Lefschetz to R. L. Moore, March 7, 1925, Box 21, R. L. Moore Archives, Center for American History, The University of Texas at Austin.
- A6. L. Barrett, personal correspondence with the author, October 19, 2002.

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So far, the twenty-first century has been very good to **DAVID E. ZITARELLI**. In addition to publishing a paper on towering figures in American mathematics and a book on the history of an MAA section, a video of his lecture on the genesis of the Moore Method was produced and distributed. In 2001 he won the Lindback Award for Excellence in Teaching and was chosen Professor of the Year by students in Temple’s Honors Program. The following year he was a Buckingham Scholar-in-Residence at Miami University in Ohio. In 2003 he was elected the first chair of HOMSIGMAA, the MAA special interest group on the history of mathematics. He has also organized AMS–MAA special sessions on the history of mathematics every year in the millennium, whether 2000 is included or not.

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Untitled poem from the collection *Crossing the Equal Sign*

Every 4:00 A.M. Substance is the first of six cats to arise and go rummaging
 through the wastebasket
 for something, anything
 to chase around the room.
 My child says he’s frightened
 frightened of everything.
 But I say Substance is looking for something to prove.
 I say give Substance definitions
 notation
 axioms.
 Yes, Substance needs some axioms
 to scratch along the floor.

——Submitted by Marion D. Cohen
 University of Sciences
 Philadelphia, PA