

1 April 2010

Homework 8

Theoretical Linear Algebra 3051

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Due Tuesday 6 April 2010.

1. Give an example of a (non-trivial) 3×3 matrix, and two different Schur decompositions for it.
2. Give an example of a space V with an inner product, and an invertible map $T \in \mathcal{L}(V)$ such that $T^{-1} \neq T^*$.
3. Give an example of an operator $T \in \mathcal{L}(V)$ such that $\|T\| = 1$, and another operator $A \in \mathcal{L}(V)$, $A \neq 0$ with $\|A\| < 1$.

4. *

Let A be self-adjoint and $v \neq 0$. Consider a Krylov subspace of dimension m , i.e., $\mathcal{K}_m(A, v) = \text{span}\{v, Av, A^2v, \dots, A^{m-1}v\}$ in \mathbb{R}^n . Show how to construct an orthonormal basis $\{v_1, v_2, \dots, v_m\}$ of $\mathcal{K}_m(A, v)$ using a three-term recurrence. Prove that indeed the way you suggest gives an orthonormal basis.