

Contents

Preface	vii
Notation	x
1 Generalized Solutions to Monge–Ampère Equations	1
1.1 The normal mapping	1
1.1.1 Properties of the normal mapping	2
1.2 Generalized solutions	6
1.3 Viscosity solutions	8
1.4 Maximum principles	10
1.4.1 Aleksandrov’s maximum principle	12
1.4.2 Aleksandrov–Bakelman–Pucci’s maximum principle	13
1.4.3 Comparison principle	16
1.5 The Dirichlet problem	17
1.6 The nonhomogeneous Dirichlet problem	20
1.7 Return to viscosity solutions	24
1.8 Ellipsoids of minimum volume	26
1.9 Notes	30
2 Uniformly Elliptic Equations in Nondivergence Form	31
2.1 Critical density estimates	31
2.2 Estimate of the distribution function of solutions	37
2.3 Harnack’s inequality	41
2.4 Notes	43
3 The Cross-sections of Monge–Ampère	45
3.1 Introduction	45
3.2 Preliminary results	47
3.3 Properties of the sections	50
3.3.1 The Monge–Ampère measures satisfying (3.1.1)	50
3.3.2 The engulfing property of the sections	55
3.3.3 The size of normalized sections	57
3.4 Notes	62

4	Convex Solutions of $\det D^2u = 1$ in \mathbb{R}^n	63
4.1	Pogorelov's Lemma	63
4.2	Interior Hölder estimates of D^2u	67
4.3	C^α estimates of D^2u	70
4.4	Notes	74
5	Regularity Theory for the Monge–Ampère Equation	75
5.1	Extremal points	75
5.2	Extremal points of solutions	77
5.3	A strict convexity result	80
5.4	$C^{1,\alpha}$ regularity	85
5.5	Examples	93
5.6	Notes	93
6	$W^{2,p}$ Estimates for the Monge–Ampère Equation	95
6.1	Approximation Theorem	95
6.2	Tangent paraboloids	99
6.3	Density estimates and power decay	101
6.4	L^p estimates of second derivatives	108
6.5	Proof of the Covering Theorem 6.3.3	112
6.6	Regularity of the convex envelope	119
6.7	Notes	122
	Bibliography	126
	Index	126