1. Let $K \subset \mathbb{C}$ be compact. Let U be a bounded component of $\mathbb{C} \backslash K$. Show that if p is a polynomial, then $\sup_{z \in U} |p(z)| \leq \sup_{z \in K} |p(z)|$.

2. Let $K_n = \{z : |z| \leq n\}$. For each $n \in \mathbb{N}$, $n > 1$, find a polynomial p_n such that

$$\left| \frac{1}{z-n} - p_n(z) \right| \leq C_n \quad \text{if } z \in K_{n-1}$$

where the C_n are such that $\sum_{n=2}^{\infty} C_n$ is finite. Let $p_1 = 0$. Show that

$$\sum_{n=1}^{\infty} \left(\frac{1}{z-n} - p_n(z) \right)$$

converges uniformly in every compact set $K \subset (\mathbb{C} \backslash \mathbb{N})$.