

1. Show that $\prod_{n=1}^{\infty} \cos(z/n)$ converges uniformly on compact subsets of \mathbb{C} to an entire function f . Find the zeros of f .
2. Do problems 9, 12, and 13, page 174, of the textbook.

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Problem 2 of Homework 4 (assertions 1-4 below hold with far weaker assumptions):

1. Let $\chi : \mathbb{R} \rightarrow \mathbb{R}$ be a C^k function with support contained in the interval $(-\varepsilon, \varepsilon)$ (here and below $\varepsilon > 0$), let I be an open interval, and let $f : I \rightarrow \mathbb{R}$ be continuous. The convolution $(f * \chi)(x)$ can be defined for x in the set

$$I_\varepsilon = \{x \in I : \text{dist}(x, \mathbb{R} \setminus I) > \varepsilon\}.$$

Show that $f * \chi$ is of class C^k on I_ε , and

$$\frac{d^\ell}{dx^\ell}(f * \chi) = f * \frac{d^\ell \chi}{dx^\ell} \quad \text{for } \ell \leq k.$$

2. Let $\chi : \mathbb{R} \rightarrow \mathbb{R}$ be continuous with support in the interval $(-1, 1)$. Let

$$c = \int \chi(x) dx.$$

Define $\chi_\varepsilon(x) = \varepsilon^{-1} \chi(x/\varepsilon)$. Let I be an open interval, $f : I \rightarrow \mathbb{R}$ continuous. Show that for any compact $K \subset I$, $(f * \chi_\varepsilon)|_K$, which is defined for all sufficiently small $\varepsilon > 0$, converges uniformly to cf .

3. Show that if ϕ and χ are continuous, nonnegative, and compactly supported, then $\phi * \psi$ is nonnegative and compactly supported.

4. Let I be an open interval, $f : I \rightarrow \mathbb{R}$ continuous, and $\chi : \mathbb{R} \rightarrow \mathbb{R}$, χ_ε as in Problem 2. If $\varphi : I \rightarrow \mathbb{R}$ is a continuous function with compact support and ε is small enough, then

$$\int_I (f * \chi_\varepsilon)(x) \varphi(x) dx = \int_I f(y) (\tilde{\chi}_\varepsilon * \varphi)(y) dy$$

where $\tilde{\chi}_\varepsilon(x) = \chi_\varepsilon(-x)$.

5. Do Problem 2 of Homework 4.