

1. Find a sequence $\{z_k\}$ of complex numbers such that $\prod z_k = 0$ and $\lim_k z_k = 1/3$, and another with $z_k \neq 0$ for all k , also giving $\prod z_k = 0$ but such that $\lim_k z_k$ does not exist.

2. Let $\{\alpha_\ell\}_{\ell=1}^\infty$ be a sequence in $(0, 1)$ with $\alpha_\ell \rightarrow 0$ as $\ell \rightarrow \infty$.

(1) Show that

$$p = \prod_{\ell=1}^{\infty} (1 - \alpha_\ell)$$

converges.

(2) Suppose $\alpha_\ell = 1/(2\ell)$, $\ell = 1, 2, \dots$. Find p .

3. Give an example of a sequence of functions $f_n : (0, 1) \rightarrow \mathbb{C}$ such that f_n converges uniformly to some $f : I \rightarrow \mathbb{C}$ but e^{f_n} does not converge uniformly to e^f .

4. Do problem 11 in pg. 174 of the textbook.