

1. Let I, J be intervals in \mathbb{R} , let $f : I \rightarrow J$ and $g : J \rightarrow \mathbb{R}$ be convex functions with g increasing. Show that $g \circ f$ is convex.

2. Let I be an open interval in \mathbb{R} , $f : I \rightarrow \mathbb{R}$ continuous. Show that f is convex iff

$$\int_I f(x)g''(x) dx \geq 0 \quad \text{for all } g \in C_c^2(I) \text{ such that } g \geq 0.$$

Here $C_c^2(I)$ is the set of C^2 functions on I such that the closure of $\{x : g(x) \neq 0\}$ is a compact subset of I .

3. Supply the details of the proof of Hadamard's Three Circles Theorem.