1. Let $G \subset \mathbb{C}$ be an open set. Show that $\mathrm{Hol}(G)$ is an integral domain if and only if G is connected.

2. Let $G \subset \mathbb{C}$ be a region, let $\mathrm{Mero}(G)$ be the set of meromorphic functions on G. Show that $\mathrm{Mero}(G)$ is a field.

3. Let $S = \{z : -\pi/2 < \operatorname{Im} z < \pi/2\}$ and define $f : S \to \mathbb{C}$ by

$$f(z) = e^{e^z}.$$

Then f has a continuous extension to $\overline{S}$ (in fact f is the restriction to S of an entire function). Show that

$$\sup\{|f(z)| : z \in \partial S\} = 1$$

and that

$$\sup\{|f(z)| : z \in S\} = \infty.$$

Why is this not a contradiction to the Maximum Modulus Theorem?

4. Let S be the set in the previous problem, let $h : S \to \mathbb{C}$ be defined by $h(z) = ie^z$. Find the image of h.

5. Let $G \subset \mathbb{C}$ be a region, $a \in G$ and $r > 0$ so that $\overline{B(a,r)} \subset G$. Suppose $f,\ g : G \to \mathbb{C}$ are holomorphic, nowhere zero in G, and $|f(z)| = 3|g(z)|$ for all $z \in \partial B(a,r)$. Show that there is $c \in \mathbb{C}$ such that $g = cf$ on G.