1. Let $a : [0,1] \to \mathbb{C}$ be defined by $a(t) = e^{2\pi i t}$, let c_0 be an arbitrarily fixed complex number. Find $c : [0,1] \to \mathbb{C}$ with $c(0) = c_0$ so that

$$f_t(z) = c(t) - \sum_{n=1}^{\infty} \frac{(-1)^n}{n\, a(t)^n} \big(z - a(t)\big)^n$$

with domain $U_t = B(a(t), |a(t)|)$ $(t \in [0,1])$ is a family of function elements giving an analytic continuation of (f_0, U_0). Hint: $df_t(z)/dz = 1/z$.

2. Let $\Omega = \{z \in \mathbb{C} : z \notin (-\infty, -1] \cup [1, \infty)\}$.

(a) Find the function $f : \Omega \to \mathbb{C}$ such that

$$f'(z) = \frac{1}{z-1} - \frac{1}{z+1}, \quad f(0) = 0.$$

(b) Show that the function element (f, Ω) admits unrestricted analytic continuation to $G = \{z \in \mathbb{C} : z \neq \pm 1\}$.

(c) Describe the complete analytic function determined by (f, Ω).