1. Let

$$T(z) = \frac{az+b}{cz+d}$$

be an arbitrary Moebius transformation (thus a, b, c, $d \in \mathbb{C}$ and $ad - bc \neq 0$).

(1) Find $T'(z)$.

(2) Show that there are α, β, γ, $\delta \in \mathbb{C}$ such that $\alpha\delta - \beta\gamma = 1$ and

$$T(z) = \frac{\alpha z+\beta}{\gamma z+\delta}$$

2. Suppose that the map T in the previous problem has a, b, c, $d \in \mathbb{R}$ and $ad - bc > 0$. Show that if $\operatorname{Im} z > 0$, then $\operatorname{Im} T(z) > 0$, in other words, T maps

$$H = \{z : \text{Im} z > 0\} \qquad (\dagger)$$

to itself. Show that the same is true for T^{-1}.

3. With (x, y) denoting the points of $\mathbb{R}^2$, let μ be the measure

$$\mu = \frac{dx\,dy}{y^2}$$

on H, the set defined in ($\dagger$). Show that the transformations in Problem **2** preserve μ.

4. Let $D = \{z : |z| < 1\}$, let $a \in D$.

(1) Show that

$$T(z) = \frac{z-a}{1-\overline{a}z}$$

maps D to D bijectively.

(2) Find all bijective holomorphic maps (biholomorphisms) from D to itself.