1. Let a and b be positive numbers. Find all numbers λ for which there is a nonzero function $u : [0, a] \times [0, b] \to \mathbb{R}$ of the form $u(x, y) = X(x)Y(y)$, with X and Y both C^2, such that

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} - \lambda u = 0 \text{ for } (x, y) \in [0, a] \times [0, b] \\ u(x, y) = 0 \text{ if } y \in [0, b] \text{ and } x = 0 \text{ or } x = a \\ \dfrac{\partial u}{\partial y}(x, y) = 0 \text{ if } x \in [0, a] \text{ and } y = 0 \text{ or } y = b \end{cases}$$

2. Let u_1 and u_2 be solutions of Problem 1 corresponding to two different numbers λ_1, λ_2. Verify that

$$\iint_{[0,a]\times[0,b]} u_1(x, y)u_2(x, y)\, dx\, dy = 0$$