

1. Assume the following: that if f is a 2π -periodic function on \mathbb{R} of class C^1 , then

$$f(x) = \sum_{k=-\infty}^{\infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} f(y) e^{-iky} dy e^{ikx} \quad \forall x \in \mathbb{R}.$$

Show that if f is C^1 , 2π -periodic, and even, then

$$f(x) = \frac{1}{\pi} \int_0^{\pi} f(y) dy + \frac{1}{2\pi} \sum_{k=1}^{\infty} \int_0^{\pi} f(y) \cos(ky) dy \cos(kx).$$