

1. Let  $u_0$  be the function defined on the interval  $[0, \pi]$  by the formula

$$u_0(x) = x(\pi - x).$$

Let  $u$  be the extension of  $u_0$  to  $[-\pi, \pi]$  as an odd function, and let  $f$  be the extension of  $u$  to  $\mathbb{R}$  as a periodic function of period  $2\pi$ .

- Find  $f(-1)$  and  $f(4)$ .
- Verify that  $f$  is differentiable at 0.
- Verify that  $f$  is differentiable at  $2\pi$ .

2. Let  $u : [0, \pi] \rightarrow \mathbb{R}$  be defined by

$$u(x) = \begin{cases} x & \text{if } x \in [0, \pi/2] \\ \pi - x & \text{if } x \in (\pi/2, \pi]. \end{cases}$$

Write down the sine series of  $u$ . Show your work.

3. Let  $W$  be the space of (real- or complex-valued)  $C^2$  periodic functions on  $\mathbb{R}$  of period  $2\pi$ . This is a vector space (over  $\mathbb{R}$  or  $\mathbb{C}$ ), and the set  $K$  of elements  $f \in W$  such that  $\partial_\theta^2 f = -3^2 f$  is a finite dimensional subspace. Find a basis for  $K$ .