1. Let $v(x) = x^2$, $x \in [0, \pi]$. Find the sine series of v. That is, compute the coefficients of the representation of v as

$$v(x) = \sum_{n=1}^{\infty} v_n \sin(nx).$$

2. Find the solution of

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0 \quad \text{in } t > 0,\ 0 < x < \pi \\ u(0,t) = 0,\ u(\pi,t) = 0 \quad \text{for } t > 0 \\ u(x,0) = x^2. \end{cases}$$

3. Find the solution of

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0 \quad \text{in } t > 0,\ 0 < x < \pi \\ u(0,t) = 0,\ \dfrac{\partial u}{\partial x}(\pi,t) = 0 \quad \text{for } t > 0 \\ u(x,0) = x - \pi/2. \end{cases}$$

4. Find $u(x,t)$ periodic in x of period 2π such that

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0 \quad \text{in } t > 0, x \in \mathbb{R} \\ u(x,0) = \sin x. \end{cases}$$