

Math 127 — Summer 2004 — Final Exam
Department of Mathematics
Temple University

August 16, 2004

Name: _____

Instructor: _____

This exam consists of 10 questions. Show all your work. **No work, no credit.** Good Luck!

Question	Points	Out of
1		10
2		10
3		12
4		10
5		12
6		10
7		10
8		8
9		12
10		10
Total		104

10 points

1. Let $\mathbf{a} = i + j$ and $\mathbf{b} = i + 2j + k$ be two vectors in \mathbb{R}^3 .

(a) Find $\mathbf{a} + 2\mathbf{b}$.

(b) Calculate $\|\mathbf{a}\|$ and $\|\mathbf{b}\|$.

(c) Find $\cos \theta$ where θ is the angle between \mathbf{a} and \mathbf{b} .

(d) Find a vector orthogonal to both \mathbf{a} and \mathbf{b} .

10 points

2. Consider the two points $P_1(1, 0, 2)$ and $P_2(-1, 2, 0)$ and the plane M described by $2x - 4y + z = 10$
- (a) Find symmetric equations of the line orthogonal to M and passing through P_1 .
- (b) Find an equation for the plane containing the point P_2 and parallel to M .

12 points

3. Find all critical points of the following function and use the second derivative test to classify them as local maxima, local minima, and saddle points.

$$f(x, y) = x^3 - 3xy + y^2 + y + 1$$

10 points

4. A particle starts at the origin with initial velocity $\mathbf{v}(0) = i - j$. Its acceleration is $\mathbf{a}(t) = k$. Find its position function. (Note that $\mathbf{r}(0) = 0$.)

12 points

5. Suppose the temperature at a point (x, y, z) in space is given by

$$T(x, y, z) = \frac{45z}{1 + 2x^2 + 3y^2},$$

where T is measured in degree Celsius and x, y, z in meters.

(a) Find the **direction** in which the temperature increase fastest at $(1, -1, 2)$.

(b) Find the maximum rate of increase at $(1, -1, 2)$.

(c) Find the equation of the tangent plane to the following surface

$$\frac{45z}{1 + 2x^2 + 3y^2} = 15 \quad \text{at } (1, -1, 2).$$

10 points

6. Given $I = \int_0^3 \int_{\sqrt{\frac{x}{3}}}^1 f(x, y) dy dx$

(a) Sketch the region of integration and change the order of integration.

(b) Evaluate I if $f(x, y) = e^{y^3}$.

10 points

7. Evaluate the triple integral:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)^2 dz dy dx$$

8 points 8. Evaluate $\iint_D 2e^{x^2+y^2} dA$ where $D = \{(x, y) : -\sqrt{1-y^2} \leq x \leq \sqrt{1-y^2}, 0 \leq y \leq 1\}$

12 points

9. Consider the field $\mathbf{F} = xy\mathbf{i} + x^2y\mathbf{j}$ and the closed triangular curve C consisting of the line segments from $(0, 0)$ to $(1, 1)$, from $(1, 1)$ to $(0, 1)$, and from $(0, 1)$ to $(0, 0)$.

(a) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Green's Theorem.

(b) Verify your result to part (a) with the direct evaluation of the appropriate path(line) integral.

10 points

10. Let $\mathbf{F}(x, y) = (3 + 2xy)\mathbf{i} + (x^2 - 3y^2)\mathbf{j}$

(a) Find a function f such that $\nabla f = F$

(b) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve given by $\mathbf{r}(t) = \langle \sqrt[3]{t}, 1+t \rangle$
 $0 \leq t \leq 1$.