Math 127 — Spring 2004 — Final Exam
Department of Mathematics
Temple University

May 6, 2004

Name: ______________________________

Instructor: ______________________________

This exam consists of 11 questions. Show all your work. **No work, no credit.** Good Luck!

Question	**Points**	**Out of**
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
Total		110

10 points

1. Let $\mathbf{a} = \langle 6, 2, -3 \rangle$ and $\mathbf{b} = \langle 7, -4, 4 \rangle$.

   (a) Find $\|\mathbf{a}\|$ and $\|\mathbf{b}\|$.

   (b) Find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$.

   (c) Find $\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

   (d) Find the area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$.

10 points 2. Let ℓ be the line with parametric equations: $x = 1 - 2t$, $y = 3t$, and $z = 2 + t$.

(a) Find two points on the line ℓ.

(b) Find a vector parallel to the line ℓ.

(c) Find a nonzero vector perpendicular to the plane that contains the line ℓ and the point $(3, 0, -2)$.

10 points

3. Let C be the curve parametrized by $\mathbf{r}(t) = (5\cos t, 2t, t^2)$ with t in $[0, 10]$. Let $\vec{F} = x^2\mathbf{i} + yz\mathbf{j} + y^2\mathbf{k}$.

(a) Find $\mathbf{r}'(t)$ and $\|\mathbf{r}'(t)\|$.

(b) Express the length of C as a definite integral. Do not evaluate the integral.

(c) Compute the work done by $\vec{F}$ on a particle that moves along C from $\mathbf{r}(0)$ to $\mathbf{r}(10)$.

10 points 4. Let $f(x, y) = x^2 + \sin(xy)$.

(a) Find all first and second order partial derivatives of f.

(b) Find $\nabla f(2, 0)$.

(c) Let $\mathbf{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$. Find $D_{\mathbf{u}} f(2, 0)$.

(d) What is the maximum rate of change of f at $(2, 0)$ and in what direction?

10 points

5. Let $f(x, y) = x^3 + 6x^2 + 3y^2 - 12xy + 9x$. Find the critical points of $f(x, y)$ and classify them as local maxima, local minima, or saddle points. Be sure to show all your steps and justify your answers.

10 points

6. Evaluate the triple integral $\iiint_E \sqrt{x^2+y^2+z^2}\,dV$, where E is the solid that lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=9$ in the first octant, (i.e., $x \geq 0, y \geq 0, z \geq 0$).

10 points

7. Both parts of this problem deal with setting up a double integral for a given region of integration.

   (a) Let D be the region bounded by the line $x = -1$ and the parabola $x = 3 - y^2$. Write the double integral $\iint_D f(x, y)dA$ as an iterated integral.

   (b) Given the double integral $\int_0^1 \int_{x^2}^{x} f(x, y)dydx$, sketch the region of integration and change the order of integration.

10 points 8. Let $\vec{F} = ye^{xy}\mathbf{i} + (xe^{xy} + 3y^2)\mathbf{j}$.

(a) Find a potential function of $\vec{F}$.

(b) Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ where C is given by $\mathbf{r}(t) = t\mathbf{i} + (t^2 - 1)\mathbf{j}$ with $0 \leq t \leq 2$.

10 points

9. Let $R = \{(x, y) : \frac{x^2}{4} + \frac{y^2}{9} \leq 1\}$, and let $\vec{F} = -\frac{4}{3}y^3\mathbf{i} + 3x^3\mathbf{j}$. Let C be the boundary of R, oriented counterclockwise.

   (a) Use Green's Theorem to rewrite the integral

   $$\oint_C \vec{F} \cdot d\vec{r}$$

   as an integral over R.

   (b) Use the substitution $x = 2r\sin\theta$ and $y = 3r\cos\theta$ to compute the double integral found in part (a).

10 points 10. Let $g(x,y,z) = x^2 + y^2 + 4z^2$ and consider the surface $S = \{(x,y,z) : g(x,y,z) = 1,\ z \geq 0\}$, oriented according to the unit normal vector field $\mathbf{n} = \frac{1}{\|\nabla g\|}\nabla g$. Let $\vec{F} = xy \sin z\, \mathbf{k}$.

(a) Compute the curl of $\vec{F}$.

(b) Use Stokes' Theorem to compute $\iint_S (\nabla \times \vec{F}) \cdot d\vec{A}$

10 points 11. Let S be the sphere $x^2 + y^2 + z^2 = 4$, let $\mathbf{n}$ be the outward unit normal of S. Let $\vec{\mathbf{F}} = (x + e^{yz})\mathbf{i} + (2y + \arctan(xz))\mathbf{j} + (3z + \ln(1 + x^2 + y^2))\mathbf{k}$.

(a) Compute $\nabla \cdot \vec{\mathbf{F}}$, the divergence of $\vec{\mathbf{F}}$.

(b) Use the Divergence Theorem to compute

$$\iint_S \vec{\mathbf{F}} \cdot \mathbf{n} dA$$

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