

Instructor _____

MATH C085

FINAL EXAM

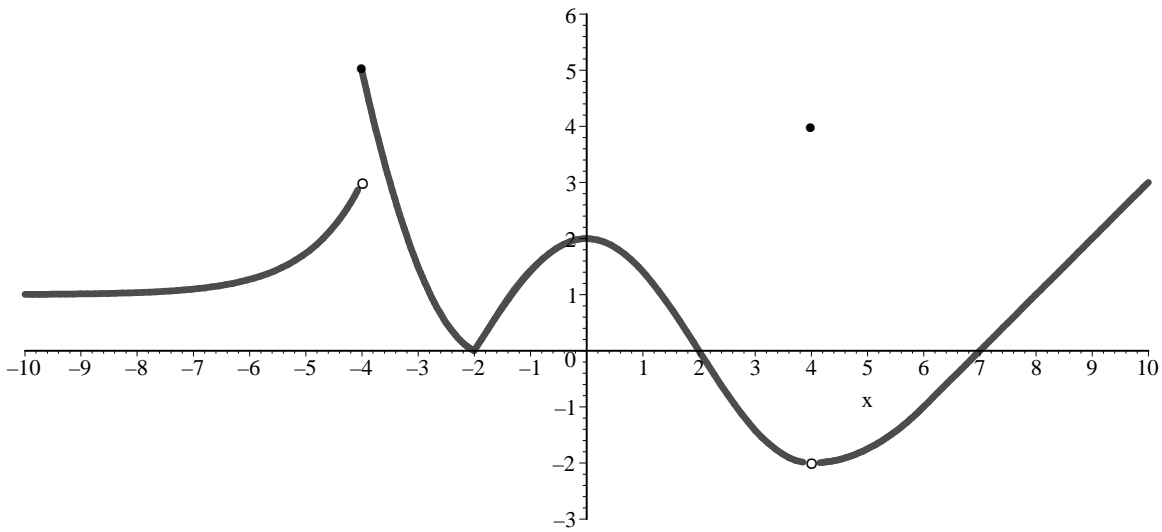
SPRING 2004

Name :

For full credit show all your work

Problem	Point Value	Score
1	16	
2	16	
3	16	
4	5	
5	8	
6	8	
7	6	
8	8	
9	6	
10	16	
Total	105	

1. (16 POINTS) The graph of a function $f(x)$ is given below.



(I) Estimate the following limits. If the limit does not exist, please explain why not.

(a) $\lim_{x \rightarrow -4} f(x)$

(b) $\lim_{x \rightarrow 4^+} f(x)$

(c) $\lim_{x \rightarrow -\infty} f(x)$

(d) $\lim_{x \rightarrow 6} \frac{f(x) + 1}{x - 6}$

(II) Is $f(x)$ continuous at $x = 4$? Explain your answer.

(III) List the numbers x at which f is not differentiable.

(IV) Does the Mean Value Theorem apply to the function $f(x)$ on the interval $[-4, 0]$? If so, find an appropriate value of c such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. If not, explain why not.

2. (16 POINTS) Find the following limits (finite or infinite). Please give exact answers.

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2}$

(b) $\lim_{x \rightarrow 0} \frac{\sin(5x)}{\sin(3x)}$

(c) $\lim_{x \rightarrow \infty} \frac{5 - 3x^2}{5x^2 + 3x - 8}$

(d) $\lim_{x \rightarrow 0^+} \frac{e^x}{\sin(2x)}$

3. (16 POINTS) Find the derivatives of the following functions.

(a) $f(x) = (x + 5)^5(x + 8)^8$

(b) $y = \sqrt{1 + e^{2x}}$

(c) $f(x) = \cos^2(3x - 2)$

(d) $g(x) = \frac{\ln x}{x^2}$

4. (5 POINTS) Use **logarithmic differentiation** to find the derivative of the function

$$f(x) = \frac{e^{3x}\sqrt{x+1}}{(x-5)^8}$$

5 (8 POINTS) The altitude of a balloon is given by the function $s(t) = 30 \arctan t$ (s in meters, t in seconds).

(a) Express the velocity and acceleration of the balloon as functions of time.

(b) Is the balloon ascending or descending at time $t = 1$?

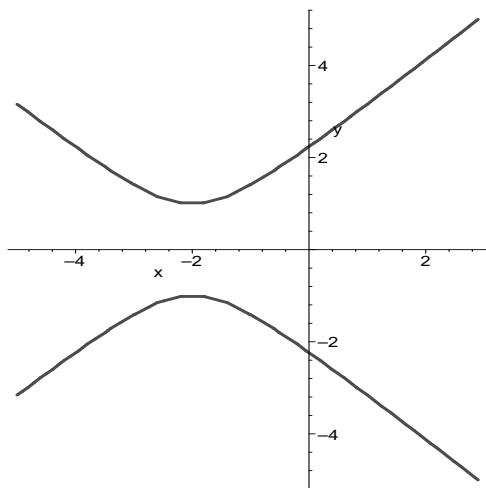
(c) Is it speeding up or slowing down at time $t = 1$?

6 (8 POINTS) Find an equation of the tangent line to the curve $y^3 - 2x^2y = x - 2$ at the point $(1, 1)$.

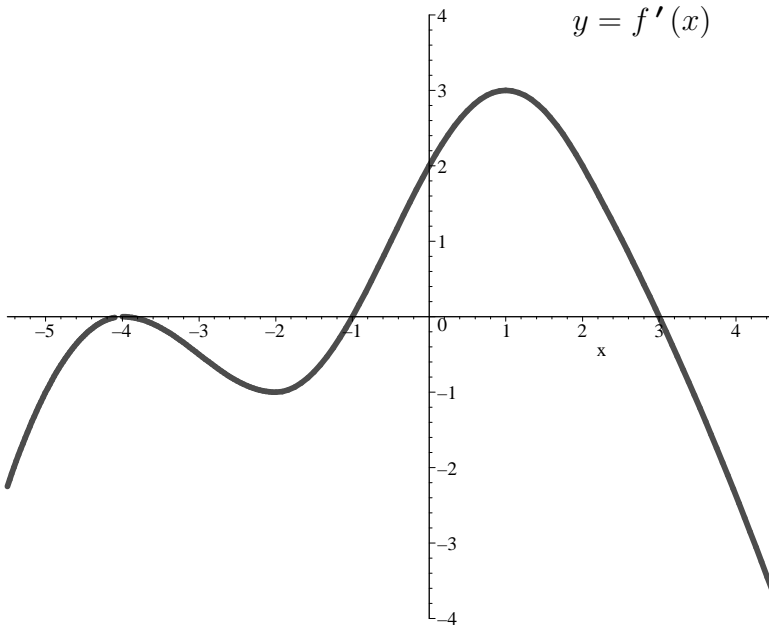
7 (6 POINTS) Use linear approximation to estimate the number $\sqrt[4]{0.9}$

8 (8 POINTS) Find the absolute maximum and absolute minimum of the function $f(x) = x^3 - 6x^2 + 15\pi$ on the interval $[-1, 3]$. Please give the exact answers.

9 (6 POINTS) On the hyperbola $y^2 = x^2 + 4x + 5$ find the point(s) closest to the origin $(0, 0)$. (Hint: minimize the square of the distance from the point (x, y) on the hyperbola to the origin.)



10 (16 POINTS) The graph of the **derivative** $f'(x)$ of a function $f(x)$ is given below. Answer the following questions about the function $f(x)$ itself and use these answers to sketch the graph of $f(x)$.



Critical numbers of $f(x)$	
Intervals of increase of $f(x)$	
Intervals of decrease of $f(x)$	
$f(x)$ has local maxima at $x =$	
$f(x)$ has local minima at $x =$	
Intervals on which the graph of $f(x)$ is concave up	
Intervals on which the graph of $f(x)$ is concave down	
The graph of $f(x)$ has inflection points at $x =$	

Sketch your graph below. Take $f(-4) = 0$. Show the tangent lines at the inflection points.

