Instructor_______________

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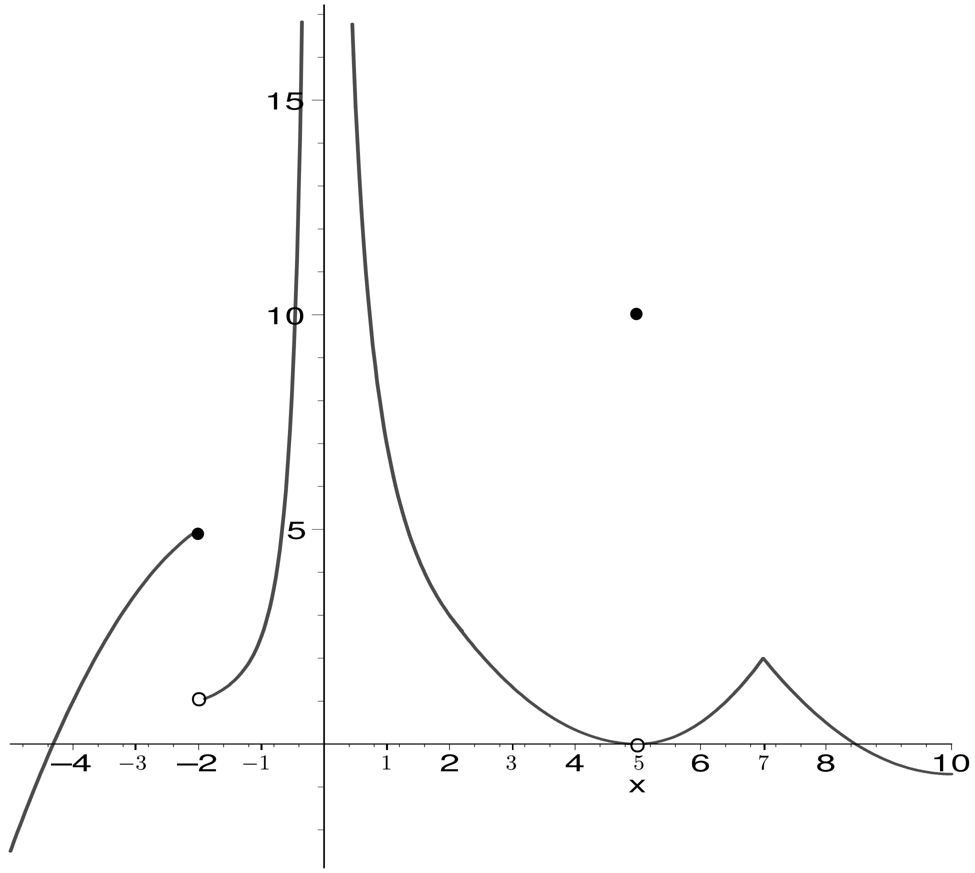
MATH C085 **FINAL EXAM** FALL 2003

Name :

For full credit show all your work.

Problem	Point Value	Score
1	16	
2	16	
3	25	
4	8	
5	6	
6	6	
7	6	
8	6	
9	16	
Total	105	

1. (16 POINTS) The graph of a function $f(x)$ is given below.



(**I**) Find the following limits (finite or infinite). If the limit does not exist, please explain why not.

(**a**) $\lim_{x \to -2^-} f(x)$

(**b**) $\lim_{x \to 5} f(x)$

(**c**) $\displaystyle \lim_{x \to 0} \frac{f(x)}{x-1}$

(**II**) Find the x-coordinates of all points of nondifferentiability of $f(x)$. Please explain briefly why $f(x)$ is not differentiable at each of these points.

(**III**) Does $f(x)$ satisfy the hypotheses of the Mean Value Theorem on the interval $[6, 8]$? If so, find all numbers c that satisfy the conclusion of the Mean Value Theorem. If not, explain why not.

2. (16 POINTS) Find the following limits (finite or infinite). Please give exact answers.

(a) $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x - 2}$

(b) $\lim_{x \to 0} \dfrac{x \sin(2x)}{(\sin x)^2}$

(c) $\lim_{x \to \infty} \dfrac{x^2 + 2x - 1}{(x-2)(2x+1)}$

(d) $\lim_{h \to 0} \dfrac{\sqrt[4]{3+h} - \sqrt[4]{3}}{h}$

3. (25 POINTS) Find the derivatives of the following functions. Please simplify your answers.

(**a**) $f(x) = (x-3)^3(x+7)^5$

(**b**) $y = \sqrt{1 + \sec^2 x}$

(**c**) $f(x) = \arctan\left(\frac{1}{x}\right)$

(**d**) $g(t) = \frac{e^{2t}}{t^2 + 1}$

(**e**) $y = x^{\tan x}$ (*Hint: use logarithmic differentiation.*)

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4. (8 POINTS) The position of a particle that is moving in a straight line is given by the function $s(t) = t^3 + 2t^2 - 4t + 8$ (s is measured in meters, t is measured in seconds).

(**a**) Express the velocity and acceleration of the particle as functions of time.

(**b**) In what direction (positive or negative) is the particle moving at time $t = 0$?

(**c**) Is it speeding up or slowing down at time $t = 0$? Please explain your answer.

5. (6 POINTS) Find an equation of the tangent line to the curve $y = (2x^3 + 1)\ln x$ at the point $(1, 0)$.

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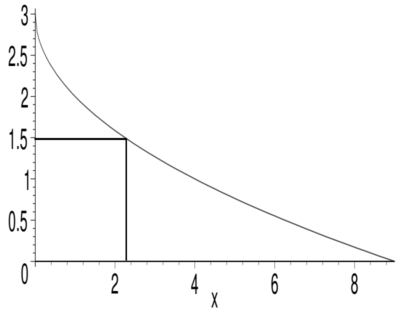
6. (6 POINTS) Use implicit differentiation to find $\frac{dy}{dx}$ where y satisfies the equation $x^3y + x = \sin(x + y)$

7. (6 POINTS) Find the absolute maximum and absolute minimum of the function $f(x) = 2\cos x + x$ on the interval $[0, \pi/2]$. Please give the exact answers.

8. (6 POINTS) Find the dimensions of the rectangle of largest area with its two sides on the x- and y-axes and one of the vertices on the curve $y = 3 - \sqrt{x}$ as in the picture below.



9. (16 POINTS) Consider a function $f(x)$ that satisfies all of the given conditions.

f has vertical asymptotes at $x = -1$ and $x = 1$.

$f'(0) = 0$.

$f'(x) > 0$ for $x < -1$ and $x > 0$, $x \neq 1$.

$f'(x) < 0$ for $-1 < x < 0$.

$f''(x) > 0$ for $x < 1$, $x \neq -1$, and $x > 3$.

$f''(x) < 0$ for $1 < x < 3$.

(**a**) Find the intervals of increase and the intervals of decrease of f.

(**b**) Find the x-coordinates of all local maxima and all local minima of f.

(**c**) Find the intervals on which f is concave up and the intervals on which f is concave down.

(**d**) Does f have any inflection points? If it does, indicate the x-coordinates of these points.

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(e) Sketch the graph of a possible function f if $f(0) = 1$, $f(3) = 2$, and $\lim_{x \to -\infty} f(x) = -1$.