GRADUATE MATHEMATICS COURSES, SPRING 2012

Math 5042: Concepts of Analysis
TR 12:30–1:50
K. Nakamura

This is the second semester of an advanced calculus course in one and several real variables. Topics include topology of metric spaces, continuity, sequences and series of numbers and functions, convergence, including uniform convergence. Ascoli and Stone-Weierstrass theorems. Integration and Fourier series. Inverse and implicit function theorems, differential forms, Stokes theorem.

Prerequisites: Permission of instructor

Math 5043: Introduction to Numerical Analysis
MWF 9:00-9:50
Prof. F. Xue

This is a one semester course which introduces the student to basic concepts in numerical analysis and scientific computing. In this discipline, numerical algorithms for the solution of a variety of problems arising in science and engineering are presented and analyzed. The goal is to study algorithms which approximate the true solution and converge to the solution in a reasonable amount of time.

Topics which will be studied include: approximation and interpolation of functions, numerical integration, finding roots of non-linear equations, numerical optimization and numerical solution of ordinary differential equations. For each topic, we will cover the problem description, basic theories and analysis of the algorithms.

In addition, we will explain how computers store and manipulate numbers, so we can study how computer arithmetic generates errors, and how the errors are propagated in the specific algorithms. Accuracy and stability of the algorithms will be studied. MATLAB projects are essential for successful completion of this course.

Textbook:

– *Introduction to Numerical Analysis, 3rd Edition*, J. Stoer and R. Bulirsch, Springer 2002.

Recommended references:

– *A First Course in Numerical Analysis, 2nd Edition*, A. Ralston and P. Rabinowitz, Dover Publications, 2001.

– *Numerical Methods in Scientific Computing, Volume 1*, Germund Dahlquist and Åke Björck, SIAM, Philadelphia, 2008.

For any questions, contact the instructor, Fei Xue, at 215.204.7588 (fxue@temple.edu).

Math 8012: Abstract Algebra II
MWF 12:00–12:50
Prof. M. Lorenz

This course, the second part of a year-long graduate-level introduction to abstract algebra, will start with a thorough discussion of field extensions. This will be followed by

Galois theory, one of the core topics of abstract algebra. The third part of the course will be devoted to rings and modules. Topics to be covered in this part include noetherian rings and modules, the structure of modules over principal ideal domains and, if time permits, an introduction to tensor products.

Textbook: Dummit & Foote: Abstract Algebra, 3rd ed., John Wiley & Sons, 2004.

Course Prerequisites: Math 8011 or equivalent or permission of instructor.

Math 8042: Real Analysis II
TR 12:30–1:50
Prof. C. Gutiérrez

This course is continuation of Math 8041 and cover the core areas of analysis and prepare students for the qualifying exam in real analysis. It focuses on the development of differentiation, abstract measures and integration, Hilbert spaces, and Hausdorff measure and fractals. Emphasis will be on exercises and problems.

Textbook: Measure and Integral, An Introduction to Real Analysis, by R. Wheeden and A. Zygmund, Marcel Dekker, 1977, ISBN: 0824764994.

Additional references:

– B. Makarov et al., Selected problems in real analysis, Translations of Math. Monographs, vol. 107, American Mathematical Society, 1992, ISBN: 0821809539. Containing many beautiful exercises and problems at different levels of difficulty.

– Real Analysis : Measure Theory, Integration, and Hilbert Spaces (Princeton Lectures in Analysis III) by Elias M. Stein and Rami Shakarchi, Princeton University Press (2005), ISBN: 0691113866.

Grading: There will be two tests, and a comprehensive final exam. The final grade will be based on the homework, tests and final exam.

Math 8052: Functions of a Complex Variable II
TR 2:00–3:20PM
Prof. B. Datskovsky

Math 8052 covers: The Riemann Mapping Theorem, Weierstrass Factorization, Entire Functions of Finite Order, Harmonic Functions and, if time allows, applications of complex analysis to Number Theory and Picard's Theorems.

The material covered in the year long sequence Math 8051-8052 provides the student with a useful background needed in the study of many areas of mathematics including number theory, several complex variables, partial differential equations, algebraic geometry and differential geometry.

Textbook:

– Functions of One Complex Variable by John B. Conway (Springer-Verlag);

Additional references:

– Complex Analysis by Lars V. Ahlfors (McGraw-Hill);

Math 8062: Differential Geometry & Topology II
MWF 11:00–11:50
C. Atkinson

This is the second semester of a year-long introduction to the geometry and topology of smooth manifolds. We will begin the fall semester with the definitions: what does it mean for a space to (smoothly) look just like $\mathbb{R}^n$? We will go on to study vector fields, differential forms (a way to take derivatives and integrals on a manifold), and Riemannian metrics.

In the spring semester, we'll study the interplay between the geometry of a manifold and certain ideas from algebraic topology. We will review the idea of the fundamental group and introduce homology – and then relate these algebraic notions to the underlying geometry. If time permits, we will talk a bit about hyperbolic manifolds – a family of manifolds where the interplay between topology and geometry is particularly strong and beautiful.

References and potential textbooks include:
– Introduction to Smooth Manifolds, by John M. Lee
– Algebraic Topology, by Allen Hatcher Three-Dimensional Geometry and Topology, by William P. Thurston

Prerequisites: Concepts of Analysis (Math 5041–5042) or equivalent and Abstract Algebra (Math 8011).

Math 8142: Partial Differential Equations
TR 11:00–12:20
Prof. S. Berhanu

A partial differential equation (PDE) is an equation involving functions and their partial derivatives. Since many processes (physical, chemical, etc) can be expressed in terms of rates of changes, PDEs appear and have applications to an enormous number of questions. For example, PDEs describe the propagation of sound and heat, the motion of fluids, the behavior of electric and magnetic fields, and the behavior of financial markets. PDEs are also crucial in understanding and solving various geometric problems.

The first semester course is intended to provide the student a basic introduction to the subject, including first-order PDEs and the three second order equations that arise in mathematical physics: the Laplace equation, the heat equation, and the wave equation. We will also cover first order equations. The solutions of these equations have different qualitative and quantitative properties and their study is essential for understanding the more general elliptic, parabolic and hyperbolic equations which will be the subject of the second semester course. The Fourier transform and Sobolev spaces and their applications to PDEs will be introduced and developed during the two semesters.

The course will be useful for students in analysis, applied mathematics, geometry, physics, and engineering.

Textbooks:
– *Partial Differential Equations*, by L. C. Evans, Graduate Texts in Mathematics vol. 19, American Mathematical Society, 1998, ISBN: 0-8218-0772-2.
– *Elliptic Partial Differential Equations of Second Order*, by D. Gilbarg and N. S. Trudinger, Springer, ISBN: 9783540411604.

Prerequisites: Basic concepts of real analysis; advanced calculus of several variables; knowledge of Lebesgue integration is useful.

Math 8210. Topics in Applied Mathematics
MWF 11:00–11:50
Prof. A. Kohlmeyer

This will be a course on high performance computing. It will include topics such as optimization of code, and issues arising in collaborative development of complex software. A detailed description will be posted here at a later time.

Math 8710: Topics in Computer Programming: High Performance Computing
TR 2:00–3:20
Prof. I. Rivin

The course will discuss algorithms and existing tools for high performance computing, especially for geometric applications. Emphasis will be given to hands-on work, including implementation on high performance graphics hardware.

The applications will be as diverse as:
1. Constructing optimal drawings of graphs in the plane and in three-dimensional space.
2. Understanding the actions of infinite groups, in Euclidean and non-Euclidean geometric settings.
3. Constructing geometric structures on three-dimensional manifolds.
4. Understanding the structure of finite and infinite groups.
5. Optimal design of portfolios of financial instruments.

The course will describe the significance of these problems in both pure mathematics and applications (for example, ”optimal drawings” come up in areas as diverse as crystallography,VLSI design, and pure topology and algebraic geometry).

We will start with an intensive introduction to computing methodology, but will start working on examples as quickly as practically possible.

The course has no formal prerequisites, but the students should be prepared for an intensive experience.

Math 9000: Topics in Number Theory
TR 12:30-1:50
Prof. M. Knopp

This will be an introduction to the theory of modular forms in a single complex variable,with the underlying group restricted to $SL(2,\mathbb{Z})$. Topics will include: the fundamental region for $SL(2,\mathbb{Z})$; theta functions; construction of modular forms by way of Eisenstein/Poincare series; Hecke operators; applications to number theory; a short introduction to vector-valued modular forms.

Prerequisites: Complex analysis.

Math 9044: Harmonic Analysis
TR 2:00-3:20
Prof. I. Mitrea

The scope of this course is to present a rapid introduction to modern tools of Harmonic Analysis which are relevant to problems in a very exciting area of mathematics, that at the interface of several branches such as Fourier Analysis, Functional Analysis, Index Theory, and Geometric Measure Theory. Topics include: properties of the Fourier and Mellin transforms, classes of domains, cone property, covering lemmas, non-tangential maximal operators, properties of Hardy-Littlewood maximal operators, the evolution of the idea of Calderón-Zygmund operator, Calderón-Zygmund theory for singular integral operators, application of this theory to boundary value problems, Muckenhoupt weights, and harmonic measure.

In a sense, the tools and techniques developed within Harmonic Analysis are at times more important than some of the results themselves. A typical paradigm is to decompose an object (function, operator, space, etc.) into pieces which are then analyzed individually and, the partial information extracted from each piece is blended into a coherent conclusion about the initial object. One historically famous example is the technique developed by

Joseph Fourier (1768-1830) who studied the properties of functions by viewing them as a linear superposition of sine and cosine curves of various amplitudes. A culmination of this principle was the development in the mid 80s of the modern theory of wavelets which has had a major impact in many branches of mathematics as well as signal processing, scattering theory, medical imaging, engineering, etc. A case in point is that FBI stores fingerprint data bases using algorithms based on wavelet theory.

The tools developed in the course will allow for an up-to-date, rigorous, and to a large extent self-contained treatment of the most basic partial differential equations in non-smooth domains. Examples include the Laplace Equation, the Lamé system of elastostatics, the Stokes system of hydrostatics, and the Maxwell system of electromagnetism.

Prerequisites: The course is appropriate for any student who has finished the graduate analysis sequence.

Math 9110. Topics in Algebra: Introduction to Noncommutative Algebras
TR 9:30–10:50
Prof. E. Letzter

Associative algebras over a field are associative (not necessarily commutative) rings equipped with a compatible vector space structure. This course will be an introduction to the study of these objects, primarily concerned with ideals and representations of finitely generated associative algebras. We will begin with fundamental results: Hilbert's Nullstellensatz (for commutative algebras), Wedderburn's Theory, finite dimensional representations, and irreducible representations. We will proceed to more advanced topics as time permits, such as algebras finite over their centers, noncommutative analogues of the Nullstellensatz, and primitive ideal theory. The course should appeal to students interested in pursuing research in algebra or related disciplines. The course should be accessible to students with a good grasp of the concepts presented in first year graduate algebra.

Math 9120. Seminar in Algebra
TR 11:00–12:20
Prof. D. Futer

This course will continue the study of mapping class groups begun in the fall semester of 2011. In particular, we will cover braid groups, the Nielsen-Thurston classification of mapping classes, and the dynamical properties of surface homeomorphisms. As in the fall semester, we derive most of our information by understanding the action of the mapping class group on Teichmüller space and the complex of curves.

Prerequisites: Permission of instructor.

Math 9410. Topics in Functional Analysis: Calculus of Variations
TR 9:30–10:50
Prof. Y. Grabovsky

The course studies both classical and vectorial problems in Calculus of Variations, exhibiting similarities and important differences. The questions of existence of solutions and of necessary and sufficient conditions for weak and strong local minima are considered.

1st Semester: Examples of variational problems. First variation, Euler-Lagrange equations. First integrals and Noether's theorem, conservation of energy, momentum, angular momentum, least action principle and link to the problem of light refraction and geodesics, ballistic motion in uniform gravity. Legendre transform, Hamiltonian formalism, canonical transformations lowering the order of the dynamical system by first integrals of motion. Connections with differential geometry, symplectic and Riemannian geometry. The eikonal and Hamilton-Jacobi equations. Weak local minima and 2nd variation. Legendre's and Legendre-Hadamard's S-condition, van Hove's sufficiency theorem. Jacobi theory of conjugate points, focal points and Morse's index form. Non-linear elasticity and Euler buckling

2nd Semester: Strong local minima, necessary conditions. Weierstrass needles, convexity and quasiconvexity, quasiconvexity at the boundary. Inner variations,Weierstrass-Erdmann corner condition, other corner conditions. Inner-outer 2nd variation. Sufficiency theorems for strong local minima. Caratheodori's "Royal Road":geodesic coverings, Caratheodori's canonical equations and Mayer bundles,Weierstrass scalar sufficiency theorem, Vectorial sufficiency theorem.Existence of solutions, Young measures, sequential w.l.s.c and quasiconvexity, Dacorogna-Marcellini's attainability theory.

Math 9420: Topics in Differential Equations: Solvability of PDEs
TR 12:30-1:50
Prof. G. Mendoza

The topic for this course will be solvability of linear partial (pseudo-)differential operators. Contrary to ordinary differential equations, where there is a very general simple theorem asserting the existence and uniqueness of solutions of such equations under rather mild conditions, the theory for partial differential equations is much more involved, and even rather tame-looking operators (for instance $\partial/\partial x + ix\partial/\partial y$) may not be solvable. The course will cover, in more or less historical order, the Ehrenpreis-Malgrange theorem, Lewy's example (the first example of a non-solvable operator), Hörmander's theorem on solvability (the first general result), Conditions (P) and (Ψ) of Nirenberg-Treves, solvability for operators of principal type, some cases of operators with characteristics of varying multiplicity, and special differential complexes. Concepts of microlocal analysis and basic symplectic geometry will be introduced as needed. This course is a gateway to many important open problems in an exciting research area.

Textbook: Notes by the instructor and articles from the literature.

Prerequisites: Math 8141 (one semester of Partial Differential Equations) or permission of instructor.