GRADUATE MATHEMATICS COURSES, FALL 2011

Math 5001: Linear Algebra (Cancelled)
TR 9:30–10:50
Instructor TBA

This a rigorous introduction to linear algebra. Vector spaces and subspaces over the real and complex numbers; linear independence and bases; linear mappings; dual and quotient spaces; fields and general vector spaces; polynomials, ideals and factorization of polynomials; determinant; Jordan canonical form. Fundamentals of multilinear algebra.

Prerequisites: Permission of instructor

Math 5041: Concepts of Analysis
TR 12:30–1:50
Instructor TBA

Advanced calculus in one and several real variables. Topics include topology of metric spaces, continuity, sequences and series of numbers and functions, convergence, including uniform convergence. Ascoli and Stone-Weierstrass theorems. Integration and Fourier series. Inverse and implicit function theorems, differential forms, Stokes theorem.

Prerequisites: Permission of instructor

Math 8011: Abstract Algebra
TR 9:30–10:50
Prof. M. Lorenz

This is a year-long graduate-level introduction to abstract algebra focusing on groups, rings, fields, and modules. The first semester will begin with a thorough treatment of groups, including the Isomorphism Theorems, the Sylow Theorems, and the basic facts concerning solvable and nilpotent groups. The second part of the semester will be devoted to rings and modules, with a view towards Galois theory (to be covered in the second semester).

Textbook: Dummit & Foote: Abstract Algebra, 3rd ed., John Wiley & Sons, 2004.

Course Prerequisites: The minimum prerequisite is Math 3098 or equivalent. However, students with a full year of undergraduate abstract algebra, and with at least one semester of undergraduate linear algebra, are more likely to succeed. Moreover, students in this course will be expected to have the mathematical maturity and experience to produce clearly written and well reasoned proofs or be able to rapidly develop these skills.

Math 8031: Probability Theory
5:00–6:20
Prof. Wei-Shih Yang

This course focus on the rigorous foundation of graduate level probability theory. Topics covered are probability measures, mathematical expectations, various convergence concepts, convergence theorems, the Law of Large Numbers, characteristic functions, the Central Limit Theorem, random walks and Markov chains.

Textbook: A Course in Probability Theory by Kai Lai Chung, Academic Press. Second edition (2000) ISBN-10: 0121741516.

Prerequisites: Math 3031 or permission of instructor.

Math 8041: Real Analysis I
TR 12:30–1:50
Prof. C. Gutiérrez

The year-long sequence 8041-8042 covers the core areas of analysis. It focuses on the development of Lebesgue measure and integration theory, differentiation, abstract measures and integration, Hilbert spaces, and Hausdorff measure and fractals. Emphasis will be on exercises and problems.

Possible Textbook:
– Measure and Integral, An Introduction to Real Analysis, by R. Wheeden and A. Zygmund, Marcel Dekker, 1977, ISBN: 0824764994.

Additional references:
– B. Makarov et al., Selected problems in real analysis, Translations of Math. Monographs, vol. 107, American Mathematical Society, 1992, ISBN: 0821809539. Containing many beautiful exercises and problems at different levels of difficulty.
– Real Analysis : Measure Theory, Integration, and Hilbert Spaces (Princeton Lectures in Analysis III) by Elias M. Stein and Rami Shakarchi, Princeton University Press (2005), ISBN: 0691113866.

Prerequisites: Basic knowledge of real variables and Euclidean topology, sequences of functions, Riemann integration.

Math 8051: Functions of a Complex Variable
TR 2:00–3:20PM
Prof. B. Datskovsky

Math 8051 covers: Elementary properties and examples of holomorphic functions; differentiability and analyticity, the Cauchy-Riemann equations; power series; conformality; complex line integrals, the Cauchy Integral Formula and Cauchy's Theorem; applications of the Cauchy Integral Formula- power series expansion for a holomorphic function, the Maximum Modulus principle, the Cauchy estimates, Liouville's Theorem; Singularities of holomorphic functions, Laurent expansions, the calculus of residues and applications to the calculation of definite integrals and sums;

zeros of a holomorphic function, the Argument Principle, Rouche's Theorem, Hurwitz's Theorem; conformal mappings.

The material covered in the year long sequence Math 8051-8052 provides the student with a useful background needed in the study of many areas of mathematics including number theory, several complex variables, partial differential equations, algebraic geometry and differential geometry.

Possible Textbook:
– Functions of One Complex Variable by John B. Conway (Springer-Verlag);

Additional references:
– Function Theory of One Complex Variable by Robert E. Greene and Steven G. Krantz (Wiley Interscience).
– Complex Analysis by Lars V. Ahlfors (McGraw-Hill);
– Princeton Lectures in Analysis II, Complex Analysis by Elias M. Stein and Rami Shakarchi (Princeton University Press).

Prerequisite: Calculus of several variables and an undergraduate level course in complex variables.

Math 8061: Differential Geometry & Topology (Schedule will change)
TR 8–9:20
Instructor TBA

This course will be an introduction to the geometry and topology of smooth manifolds. We will begin the fall semester with the definitions: what does it mean for a space to (smoothly) look just like $\mathbb{R}^n$? We will go on to study vector fields, differential forms (a way to take derivatives and integrals on a manifold), and Riemannian metrics. In the spring semester, we'll study the interplay between the geometry of a manifold and certain ideas from algebraic topology. We will review the idea of the fundamental group and introduce homology – and then relate these algebraic notions to the underlying geometry. If time permits, we will talk a bit about hyperbolic manifolds – a family of manifolds where the interplay between topology and geometry is particularly strong and beautiful.

References and potential textbooks include:
– Introduction to Smooth Manifolds, by John M. Lee
– Algebraic Topology, by Allen Hatcher Three-Dimensional Geometry and Topology, by William P. Thurston

Prerequisites: Concepts of Analysis (Math 5041–5042) or equivalent and Abstract Algebra (Math 8011).

Math 8141: Partial Differential Equations
TR 11:00–12:20
Prof. S. Berhanu

A partial differential equation (PDE) is an equation involving functions and their partial derivatives, and since many natural laws can be expressed in terms of rates of changes, PDEs appear and have applications in an enormous number of questions. For example, PDEs describe the propagation of sound and heat, the motion of fluids, the behavior of electric and magnetic fields, and the behavior of financial markets. In the first semester the course will focus in the study of three second order equations that contain the ideas and the germ of generality to study more general PDEs: the Laplace equation, the heat equation, and the wave equation. We will also cover first order equations. The solutions of these equations have different qualitative and quantitative properties and their study is essential to understand elliptic, parabolic and hyperbolic equations. The emphasis will be on ideas and techniques presented in a way that can be use later to deal with more difficult situations such us nonlinear equations. These extensions will be the subject of the second semester.

The course will be useful for students in analysis, applied mathematics, and engineering.

Textbooks:

– Partial Differential Equations, by L. C. Evans, Graduate Texts in Mathematics vol. 19, American Mathematical Society, 1998, ISBN: 0-8218-0772-2.

– Elliptic Partial Differential Equations of Second Order, by D. Gilbarg and N. S. Trudinger, Springer, ISBN: 9783540411604.

Prerequisites: Basic concepts of real analysis; knowledge of Lebesgue integration is useful but not required.

Math 8200. Topics in Applied Mathematics: Mathematical Modeling.
MWF 11:00–11:50
Y. Grabovsky, B. Seibold, and D. Szyld

Course Description: Students work in groups on projects which arise in applications from industry or science, usually posed by an outside partner. Projects are formulated in non-mathematical language, and final research results need to be formulated in a language accessible to the industry or science partner.

Course Goals: Students learn modeling by doing. On the way, they train a wide variety of skills, such as creative modeling, literature and internet research, group discussions, applied analysis, scientific programming, and presentation skills. Each student will be a master of his project, and in addition understands and learns about the other students' projects.

Grading: The course grading is based on the weekly presentations, the project reports, and on class participation. The are no exams. More than in traditional lectures, the success of the project research depends on the students' work. We expect each student to devote at least 10 hours per week to the project work.

Visit `http://www.math.temple.edu/research/applied/modeling/` for additional information.

Math 8700: Topics in Computer Programming: High Performance Computing (Cancelled, but expected to run Spring 2012)
TR 2:00–3:20
Prof. I. Rivin

The course will discuss algorithms and existing tools for high performance computing, especially for geometric applications. Emphasis will be given to hands-on work, including implementation on high performance graphics hardware.

Math 8985: Teaching in Higher Education (2 credit version)
R 3:20–5:00
Prof. Maria Lorenz

This course is required for any student seeking Temple's Teaching in Higher Education Certificate. The course focuses on the research on learning theory and the best teaching practices, with the aim of preparing students for effective higher education teaching. All educational topics will be considered through the lens of teaching mathematics and quantitative thinking.

Course Materials:

– Davis, B. G. (2009). *Tools for teaching* (2nd ed.). San Francisco: Jossey-Bass.
– "Teaching in Higher Education Seminar" - Jossey-Bass Custom Reader (ISBN 9780470568002).
– Scanned documents in Blackboard.

This is variable credit course that can be taken more than once for a total of three credits (1+2). For Fall 2011 the course will run with two credits. Please visit

Math 9005: Combinatorial Mathematics
TR 11:00–12:20
Intructor TBA

Imagine we build a structure by connecting rigid bodies by fixed-length bars. How many bars (and which) will it take to produce a structure that doesn't wobble? Given a graph G, is it the edge-disjoint union of 6 spanning trees? Are there simple algorithms to solve this problem?

Surprisingly, all these questions are very closely related. This class will explore, in detail, the geometric behavior of structures defined by length, direction, angle and other constraints, and the combinatorics that, very often, determines the geometry.

Prerequisites: Students should be comfortable with linear algebra, basic differential geometry, graph theory, and have some exposure to algorithms.

Math 9041: Functional Analysis
TR 12:30–1:50
Prof. I. Pesenson

Content:

1. Linear spaces; normed spaces; Banach spaces. Basic examples.
2. Hilbert spaces and orthonormal bases. Fejer's Theorem and Fourier series in $L_2[-\pi, \pi]$.

3. Bounded linear operators in Banach spaces. Banach- Steinhaus and Banach theorems.

4. Linear functionals, dual spaces and the Hahn-Banach theorem. Examples of linear functionals in basic function spaces.

5. Compact operators.

6. Self-adjoint bounded operators and their spectral decomposition.

7. Unbounded self-adjoint and symmetric operators and their spectral decomposition.

8. Basics of distributions and distributional Fourier transform.

9. The scale of Sobolev spaces $H^s(\mathbb{R}^n)$, $-\infty < s < \infty$, with applications to elliptic differential operators with constant coefficients.

Textbooks:

– Lusternic & Sobolev, Elements of Functional Analysis.

– W. Rudin, Functional Analysis.

Prerequisites: Real Analysis 8042.

Math 9100: Topics in Algebra
TR 9:30–10:50
Prof. V. Dolgushev

This is a one-semester course which can be roughly divided into three parts. In the first part one introduces basic ingredients of deformation quantization: star-products, Poisson structures, and Hochschild complexes. The second part is devoted to differential graded Lie algebras, Maurer-Cartan elements, and the Goldman-Millson theorem. In the third part one constructs a free resolution of an arbitrary differential graded Lie algebra and give a proof of the famous Kontsevich's formality theorem. The main goal of this course is to prepare students to work on research problems in deformation quantization.

Prerequisites: Abstract Algebra (Math 8011–8012). Students are also expected to know the definition of Lie algebra and a few simple examples of Lie algebras.

Math 9300: Seminar in Probability (Cancelled)
TR 11:00–12:20
Prof. J. Galambos

Open problems in extreme value theory, characterizations of probability distribution, and probabilistic number theory. The instructor will introduce a topic in two or three lectures, pose problems and then a volunteer from the class would read and present related results from the literature.

Prerequisite is good knowledge of concepts of probability theory.

No textbook would be followed, but books and papers will be recommended as the course progresses.

Math 9400. Topics in Analysis: Nonlinear Differential Equations
TR 2:00–3:20
Prof. C. Gutiérrez

Fully nonlinear pdes appear in several areas within Mathematics and in applications in broader scientific disciplines such as fluid dynamics, phase transitions, mathematical finance, geometric optics, and image processing in computer science. In the past few decades, there have been many new developments in this area including the understanding of regularity of generalized solutions, the study of singularities and symmetric properties of solutions. A goal in this course is to present some of these important developments including an introduction to Monge-Ampère (MA) type equations and its applications to geometric optics. These are, in general, equations involving the Jacobian determinant of a map, and arise in the mathematical description of numerous optical, acoustic, and electromagnetic applications, in particular, in lens and reflector antenna design. The course will present basic facts about the MA equation such us weak solutions, existence, uniqueness and describe regularity results. We next build up on these ideas to show how to construct generalized solutions to other problems by means of the Minkowski method from convex geometry and also by optimal mass transportation. The physical background underlying some of these problems is related to the Maxwell equations that will described in detail.

The course would be useful for graduate students interested in analysis, applied mathematics, physics and engineering.

Prerequisites: Knowledge of real analysis and basic PDEs.

References:

– M. Born and E. Wolf. *Principles of Optics, Electromagnetic theory, propagation, interference and diffraction of light*. Cambridge University Press, seventh (expanded), 2006 edition, 1959.

– D. Gilbarg and N. S. Trudinger *Elliptic Partial Differential Equations of Second Order*. Springer-Verlag, second edition, revised 3rd printing 1998.

– C. Villani. *Optimal transport, old and new*. Optimal transport, old and new. Grundlehren der mathematischen Wissenschaften, Vol.338, Springer-Verlag, 2009. Available for download at `http://math.univ-lyon1.fr/homes-www/villani/`

– C. E. Gutiérrez. *The Monge–Ampère equation*. Birkhäuser, Boston, MA, 2001.

Math 9011. Homological Algebra
Time to be determined
Prof. M. Lorenz

This is a one-semester course which can be roughly divided into four parts. The first part is devoted to chain complexes of modules over a ring. In this part we introduce operations on chain complexes, the notion of quasi-isomorphism and the notion of chain homotopy. We also discuss Snake type lemmas, filtered complexes, mapping cones and cylinders. In the second part we introduce rudiments of category theory: categories, functors, natural transformations, adjoint functors, and (co)limits. In this part we also introduce the notion of Abelian category and discuss examples. In the third part we introduce the general theory of derived functors together with the classical examples Ext and Tor. In this part we also talk about Hochschild (co)homology and group (co)homology. The last part is devoted to spectral sequences and their applications.

Prerequisites: Permission of instructor.

Math 9120. Seminar in Algebra
Time to be determined
Prof. D. Futer

This course will focus on the symmetry groups of manifolds and cell complexes. We will look at both the geometric and algebraic properties of the mapping class group, which can roughly be thought of as the group of symmetries of a surface. One of the ways in which we will study the group is by designing a certain cell complex on which it acts by isometries. This principle has wide generalizations: one can derive a great deal of information about a group by constructing a geometric object for the group to act on.

Prerequisites: Permission of instructor.

Math 9410. Topics in Functional Analysis: Calculus of Variations
TR 9:30–10:50
Prof. Y. Grabovsky

The course introduces basic ideas of classical Calculus of Variations and its connection to physics, mechanics and engineering. Many modern areas of mathematics take their origin in calculus of variations. The course will cover the following topics.

Semester 1: First variation and Euler-Lagrange equations. Null-Lagrangians and the Caratheodory's "Royal Road". Geodesic coverings, the eikonal and the Hamilton-Jacobi equation. Second variation and Jacobi's theory of conjugate points. Strong variations and Weierstrass E-function. Hamiltonian formalism and convex duality. Hilbert's invariant integral.

Semester 2: Variational problems with variable end-points. Second variation and transversality. Focal points and Morse index. Variational problems with no-solutions and the problem of existence. Weak lower semi-continuity and the direct method in calculus of variations. Young measures as generalized curves. Ill-posed variational problems and relaxation. Introduction to the problem of multiple integrals in calculus of variations. Applications: non-linear elasticity, shape memory effect. Questions of regularity.

Prerequisites: Advanced Calculus, Linear Algebra

Math 9420: Topics in Differential Equations: Microlocal Analysis
TR 11:00-12:20
Prof. G. Mendoza

We plan a two semester sequence on linear partial differential operators. The first semester will be primarily devoted to microlocal analysis, and includes the general theory of pseudodifferential and Fourier integral operators. The semester will begin with a review of distribution theory, the Fourier transform, and Sobolev spaces, and continue with an analysis of singularities of distributions (wave-front set). The theory of pseudodifferential operators will then be presented and extended to Lagrangian distributions (from which the theory of Fourier integral operators follows). Other technical topics will include continuity of pseudodifferential operators. Among the applications of pseudodifferential operators we will discuss the theory of elliptic operators on compact manifolds including a brief introduction to the Atiyah-Singer Index Theorem, elliptic boundary value problems, and local (and microlocal) solvability. Applications of Fourier integral operators include the Cauchy problem from hyperbolic operators such as the wave operator.

The second semester will be devoted to recent developments in the analysis of certain classes of differential operators on noncompact manifolds.

Textbook: Notes by the instructor and selected sections of the references below.

References:

– Duistermaat, J., Fourier integral operators.
– Chazarain, J. and Piriou, A., Introduction to the theory of linear partial differential equations.
– Hörmander, L., The analysis of linear partial differential operators. Vol. I. Distribution theory and Fourier analysis.
– Melrose, R., The Atiyah-Patodi-Singer index theorem.
– Treves, F., Introduction to pseudodifferential and Fourier integral operators. Vol. 1: Pseudodifferential operators; Vol. 2: Fourier integral operators.

Prerequisites: Math 8141–8142 (Partial Differential Equations) or permssion of instructor.